UMass Boston
Math 260, Summer 2012

Exam #2
August 23, 2012

Name: .... Student ID: ....

Directions:

(1) PRINT your name on the blue book and on this test on the designated spot.

(2) This test is closed-book, no-calculator (don't panic, you won't need one!)

(3) Partial credit may be given for an incomplete answer, but NO CREDIT is given for an answer that: is not relevant to the question asked, OR does not give an explanation, OR does not show the computational work, OR cannot be read or understood. (Be wise: keep the grader happy!)

(4) Explanations should be *succinct*: "brief and to the point."

(5) This test lasts 90 minutes and is worth 100 points.

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(1) Compute the following determinants:

$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} \quad , \quad \begin{vmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix}$$

(2) Let

$$A = \begin{bmatrix} -3 & 0 & 2 & 0 \\ 0 & -1 & -2 & 2 \\ -2 & 0 & 2 & -1 \\ -2 & 0 & 2 & -1 \end{bmatrix}$$

(a) Show that $\lambda = -1$ is an eigenvalue of A.
(b) Find a basis of the eigenspace associated to the eigenvalue $\lambda = -1$.
(c) Find the geometric multiplicity of the eigenvalue $\lambda = -1$.
(d) Find another eigenvalue $\lambda \neq -1$ of A.

(3) Let

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

(a) Find a matrix S and a diagonal matrix D such that $A = SDS^{-1}$.
(b) Compute A^{2012}.
(c) Is there any orthogonal matrix S such that $A = SDS^{-1}$?

(4) Let V be the linear subspace of $\mathbb{R}^3$ spanned by

$$w = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(a) Find a basis of $V^\perp$, the orthogonal complement of V.
(b) Find an orthonormal basis of $V^\perp$.
(c) Find the matrix of the orthogonal projection of $\mathbb{R}^3$ onto $V^\perp$.
(d) Is there any orthogonal transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(w) = e_3$?
(e) Find an orthogonal transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(w) = \sqrt{3}e_1$.

(5) Consider the matrix

$$A = \begin{bmatrix} 8 & -2 \\ -2 & 5 \end{bmatrix}$$

(a) Show that $w = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is an eigenvector and find the corresponding eigenvalue.
(b) Find an orthonormal eigenbasis for A.
(c) Sketch the curve $8x^2 - 4xy + 5y^2 = 1$.

(6) Let V be the set of 3×3 skew-symmetric matrices ($A^T = -A$).
(a) Show that V is a linear subspace of M_3, the space of all 3×3 matrices.
(b) Find a basis $\mathcal{B}$ of V and find the dimension of V.
(c) Show that if A and B are matrices in V, then $AB - BA$ is also in V.

(7) Let V be the linear space of 3×3 skew-symmetric matrices ($A^T = -A$) and $T\colon V \to V$, $T(X) = AX - XA$, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) Show that T is a linear transformation.
(b) Find the matrix of T in the basis $\mathcal{B}$ (found above).