

Collection of Typical Final Examination Problems

Math 115 College Algebra

Chapter 1: Linear Equations and Inequalities

1. Solve for s . $\frac{D}{r} + \frac{D}{r+s} = t$
2. (a) Which expressions are equivalent to $-\frac{x+7}{y}$?
 - a. $\frac{x+7}{-y}$ b. $\frac{-x-7}{-y}$ c. $\frac{-x-7}{y}$
- (b) Which expressions are equivalent to $-\frac{3w}{-x-y}$?
 - a. $\frac{3w}{x+y}$ b. $\frac{-3w}{-x-y}$ c. $-\frac{3w}{x+y}$

Chapter 2: Linear Equations, Functions

3. Given the equation $7x - 2y = 4$, find the slope and y -intercept. Then graph the equation.
4. Write an equation of the line that contains the point $(-2, 5)$ and is perpendicular to a line with slope $\frac{1}{2}$. Write the answer in slope-intercept form or standard form. Then graph the equation and label both intercepts with their coordinates.
5. Write an equation of the line perpendicular to the line $2x + y - 4$ and passing through the point $(2, 6)$. Write the answer in slope-intercept form.
6. True or false? $f(x)$ is a value of x , the independent variable.
7. Find the domain and range: $f(x) = \sqrt{2x-1}$.
8. For $f(x) = \sqrt{\sqrt{x-5}-2}$.
 - (a) Find the domain of f .
 - (b) For which x does $f(x) = 3$?
9. For $f(x) = \sqrt{2-|x-5|}$.
 - (a) Find the domain of f .
 - (b) For which values of x does $f(x) = 1$?

Systems of Linear Equations

10. Solve the system by using the addition method. Then write the solution as an ordered pair and check your answer.

$$2x + 5y = 9$$

$$4x - 7y = -16$$

11. Solve the system.

$$4x - 2y = 7$$

$$-3x + 5y = 0$$

12. Solve the system.

$$x = 1 - 2y$$

$$7y = 3x - 16$$

Polynomials

13. Perform the indicated operations and simplify.

(a) $(x-1)(x+1) - (x-4)(x+4)$

(b) $(x-1)^2 - (x-4)^2$

14. Multiply.
- $(x-1-\sqrt{3})(x-1+\sqrt{3})$

15. Perform the indicated operations and simplify.

(a) $4x(3x^2-7) - (2x^2+9)(6x-1)$.

(b) $(2x-3)(x-4) - (x-5)^2$.

16. Perform the indicated operations and simplify.

$(a+b-c)(a+b+c) - (b+c)(b-c)$

17. Multiply and express the result in scientific notation and in standard notation.

$$\frac{(3.1 \times 10^{-6})(9 \times 10^7)}{1.2 \times 10^3}.$$

18. Simplify and write the answer with positive exponents only.

$$\left(\frac{-6a^2b^{-3}}{5a^{-1}b} \right)^{-2}$$

19. Simplify the expression, and write the answer with positive exponents only.

$$\frac{(3^{-1}xy^{-3})^{-3}(x^{-4}y)}{(x^0y^5)^{-1}}$$

20. Factor completely.
- $8a^2 - 50$

21. Factor.
- $3x^2 - 24x + 48$

22. (a) Factor completely.
- $15x^2 - 31x - 24$

(b) Find the x -intercepts of the function $g(x) = 15x^2 - 31x - 24$.

(c) What is the y -intercept?

23. Solve for
- x
- :
- $35x^2 + 16x = 3$
- .

24. True or false.

(a) $(a+b)^2 = a^2 + b^2$,

(b) The function $f(x) = x^2 + \frac{1}{x^2}$ has equivalent form $f(x) = \left(x - \frac{1}{x}\right)^2 + 2$, proving that the range of f is $[2, \infty)$.

Rational Expressions and Rational Equations

25. Multiply these two rational expressions and simplify the result.

$$\left[- \left(\frac{2 - 3x + x^2}{2 - x} \right) \right] \left[\frac{3 - x}{9 - x^2} \right]$$

26. Multiply and simplify the result. $\frac{a + b}{a - b} - \frac{a - b}{a + b}$

27. Add these two rational expressions and simplify the result.

$$\frac{-10}{z^2 - 6z + 5} + \frac{15}{z^2 - 4z - 5}$$

28. Solve the equation. Be sure to check all answers.

$$\frac{x}{x + 6} = \frac{72}{x^2 - 36} + 4$$

29. Solve for x .

$$\frac{9}{x - 4} + \frac{3}{x + 2} = \frac{6}{x^2 - 2x - 8}.$$

Radicals

30. (a) Which of the following expressions are negative? Which are equal to each other?

$$(-8)^2, \quad -8^2, \quad \sqrt{(-64)^2}$$

- (b) Which of the following expressions are negative? Which are equal to each other?

$$\sqrt[3]{-8}, \quad -\sqrt[3]{8}, \quad -\sqrt[6]{64}$$

31. Find the domain. $f(x) = \sqrt{2x-1}$.

32. (a) Evaluate the function values $f(-3)$, $f(0)$, $f(5)$, and $f(12)$ for $f(x) = \sqrt{x+4}$. Is it possible to evaluate $f(-5)$ over the real numbers?

- (b) Write the domain of f in interval notation. State the exact coordinates of the endpoint of the graph of this equation.

- (c) Plot these five points and use them to draw a rough sketch of the graph.

- (d) For which x is $f(x) = 11$?

33. Simplify the expressions $(-8)^{-2/3}$ and $(-\frac{1}{3}) \cdot (-\frac{1}{3})^{-1/3}$

34. Simplify the expression $\sqrt{16^{-3/4} \cdot 16^{5/4} \cdot 16^{1/2}}$.

35. Add or subtract as indicated, simplifying as much as possible.

$$8\sqrt{3} - 2\sqrt{27} + \sqrt{75}.$$

36. Rationalize the denominator of each expression. $\frac{8}{\sqrt[3]{2}}$, $\frac{4}{\sqrt[3]{4}}$, $\frac{18}{\sqrt[3]{6}}$

37. Simplify the expression: $\frac{(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})}{x^2 - y^2}$.

38. Find an equivalent expression that is in the most simplified radical form.

$$\sqrt{\frac{8}{\sqrt[3]{2}}}.$$

39. Consider the expression: $\frac{\sqrt{2x} - \sqrt{3}}{\sqrt{2x} + \sqrt{3}} + \frac{\sqrt{2x} + \sqrt{3}}{\sqrt{2x} - \sqrt{3}}$.

- (a) Evaluate the expression for $x = 6$ with answer a rational number (a fraction).

- (b) Simplify the expression when x is not specified. The answer will be a rational expression in x .

40. Solve. $\sqrt{8x+5} = \sqrt{2x} + 2$

41. Solve. $|x+3| = \sqrt{x+15}$

42. Solve. $\frac{\sqrt[3]{x+28}}{\sqrt[3]{x-28}} = 2$.

Quadratic Equations and Functions

43. Find the value of n so that the expression is a perfect square trinomial. Then factor the trinomial $x^2 + x + n$, using the value of n you found.

44. Simplify the expression. $\frac{18 + \sqrt{180}}{3}$

45. Solve the equations $2x^2 - 3x = 5$ and $32z^2 - 20z = 3$.

46. Solve for x . $x - \frac{1}{8}x^2 = 2$.

47. An object is fired straight up into the air from an initial height of 384 ft with an initial velocity of 160 ft/sec. The height in feet is give by

$$h(t) = -16t^2 + 160t + 384$$

where t is the time in seconds after the launch.

- (a)
 - i. Find the height of the object after 3 sec.
 - ii. Find the height of the object after 7 sec.
 - iii. Find the time required for the object to hit the ground.
 - (b)
 - i. Find the time when the object attains its maximum height.
 - ii. Find that maximum height.
 - (c) Was the object ever again at a height of 384 ft and, if so, when?
48. Given the equation

$$x^2 - 14x + 1 = 0.$$

- (a) Solve by completing the square and using the square root property, leaving the answers in simplified radical form (no calculators permitted):
 - (b) Solve by using the quadratic formula.
 - (c) Which method did you find easier to use?
49. Given $f(x) = x^2 + 6x + 5$.
- (a) Determine the x -intercepts.
 - (b) Determine the y -intercept.
 - (c) Determine the vertex of the parabola.
 - (d) Find the point of which the value of y is the same as at the y -intercept. (This point is called the symmetric point of the y -intercept.)

Word Problems.

50. The sum of three consecutive odd integers is 41 less than four times the largest. Find the integers.

Check your answer.

51. The square of an odd integer less the next greater odd integer equals 54. Find the value of the odd integer being squared.

52. A driver makes the trip from Boston to Hartford at an average speed of 40 mph and covers the same distance back at an average speed of 50 mph. The total time for both trips was four-and-a-half hours.

- (a) How long did the trip to Hartford take?
(b) What is the distance from Boston to Hartford?

Check your answers. Use of a calculator is suggested.

53. Shawna has money distributed between two accounts: an account that earns 5% simple interest and an account that earns 3.5% simple interest. She has \$100 less invested at 3.5% than at 5%. If after 1 year her total interest is \$81.50, how much did she invest at 5%?

Check your answers on a calculator.

54. A driver travels 30 miles to the gym at a constant speed and travels the same distance home at a constant speed. His speed on the return trip was sixteen miles per hour faster, and the total time for both trips was 2 hrs. Find his speed on the way to the gym.

Check your work.

55. The length of a rectangle is 3 feet longer than a side of a square while the width of the rectangle is 5 feet shorter than a side of that square. The area of the square is 45 square feet greater than the area of the rectangle. Find the length of a side of that square.

56. Determine the length and width of a rectangle with a perimeter of 34 ft and an area of 60 ft^2 .

57. Determine the length of the radius of a circle whose area is numerically one-third of its circumference.

58. Suppose that in the next presidential election, Donald Trump gets nine-tenths of the Republican vote and one-fifth of the votes of non-Republicans, and his total vote percentage in the popular vote turns out to be 48%.

What fraction of those voting were Republicans?

Answers are on the following pages.

Answers.

1. $s = \frac{2Dr - tr^2}{tr - D}$.
2. (a) a. and c. (b) all of them
3. $m = 7/2$, y -intercept is $(0, -2)$. Find another point such as $(2, 5)$ and connect these two points with a straight line (graph omitted).
4. $y = -2x + 1$ or $2x + y = 1$. x -intercept: $(\frac{1}{2}, 0)$, y -intercept: $(0, 1)$. Find a third point, say $(1, -1)$ and draw a straight line through these 3 points.
5. $y = \frac{1}{2}x + 5$.
6. False; $f(x)$ is a value of y , the dependent variable.
7. The domain is $\left[\frac{1}{2}, \infty\right)$ and the range is $[0, \infty)$.
8. (a) $[9, \infty)$ (b) $x = 126$.
9. (a) $[3, 7]$ (b) $x = 4$ or $x = 6$.
10. $(-\frac{1}{2}, 2)$
11. $\left\{\left(\frac{5}{2}, \frac{3}{2}\right)\right\}$.
12. $\{(3, -1)\}$.
13. (a) 15 (b) $6x - 15$
14. $x^2 - 2x - 2$
15. (a) $2x^2 - 82x + 9$
(b) $x^2 - x - 13$
16. $a^2 + 2ab$
17. 2.325×10^{-1} or 0.2325
18. $\frac{25b^8}{36a^6}$
19. $\frac{27y^{15}}{x^7}$
20. $2(2a - 5)(2a + 5)$
21. $3(x - 4)^2$
22. (a) $(3x - 8)(5x + 3)$ (b) $8/3$ and $-3/5$ (c) -24
23. $x = 1/7$ or $x = -3/5$.
24. (a) False. (b) True.
25. $\frac{x - 1}{x + 3}$
26. $\frac{4ab}{a^2 - b^2}$
27. $\frac{5}{z^2 - 1}$ or $\frac{5}{(z - 1)(z + 1)}$.
28. $x = 4$, giving $\frac{4}{10} = \frac{72}{-20} + 4$, then $0.4 = -3.6 + 4$, which checks. A potential answer of $x = 6$, a solution to the resulting quadratic, is inadmissible.

29. $x = 0$.
30. (a) only -8^2 is negative and the first and third expressions are equal.
(b) All three are negative and equal.
31. $[\frac{1}{2}, \infty)$
32. (a) The four values are: 1, 2, 3, 4. $f(-5)$ cannot be evaluated as a real number.
(b) $[-4, \infty)$. The endpoint is at $(-4, 0)$,
(c) Graph omitted. It starts at $(-4, 0)$, passes through $(0, 2)$ and continues in the first quadrant. The shape is a half-parabola with the axis of symmetry the x -axis.
(d) $x = 117$.
33. $1/4$ and $\sqrt[3]{3}/3$.
34. 4.
35. $7\sqrt{3}$.
36. $4\sqrt[3]{4}$, $2\sqrt[3]{2}$, $3\sqrt[3]{36}$
37. $\frac{1}{x+y}$.
38. $2\sqrt[3]{2}$.
39. (a) $10/3$ (b) $\frac{4x+6}{2x-3}$
40. $\{\frac{1}{18}, \frac{1}{2}\}$
41. $x = -6$ or $x = 1$.
42. $x = 36$.
43. $n = \frac{1}{4}$ and $x^2 + x + \frac{1}{4}$ factors as $(x + \frac{1}{2})^2$.
44. $6 + 2\sqrt{5}$.
45. $x = -1$ or $x = 5/2$; and $x = -1/8$ or $x = 3/4$.
46. $x = 4$.
47. (a) i. 720 ft
ii. 720 ft
iii. 12 sec
(b) i. 5 sec
ii. 784 ft
(c) Yes, at time 10 sec.
48. (a) $7 \pm 4\sqrt{3}$ (b) $7 \pm 4\sqrt{3}$ (c) Completing the square.
49. (a) $(-1, 0)$, $(-5, 0)$
(b) $(0, 5)$
(c) $(-3, -4)$
(d) $(-6, 5)$
50. The integers are 31, 33, and 35.
Check: $(31 + 33 + 35) = 4(35) - 41$, which is $99 = 99$. It checks.
51. The odd integer being squared must be -7 . Minus seven squared is $+49$, then subtract -5 to get 54. (Eight solves the equation but it is not odd.)

52. (a) Two-and-a-half hours. (b) 100 miles.
53. 1,000. Check: $0.05(1000) + 0.035(900) = 50 + 31.50 = 81.50$.
54. 24 miles per hour.
Check: $\frac{30}{24} + \frac{30}{24+16} = 1.25 + \frac{30}{40} = 1.25 + 0.75 = 2$. It checks.
55. The length of a side of that square is 15 feet.
56. The length is 12 ft and the width is 5 ft.
57. The length of the radius is $2/3$ of a unit.
58. Two out of five. That is: $2/5$, 0.40, or 40% of the voters were Republicans.