Sample Final Examination

Math 115 College Algebra
(Exam on May 24, 2019)

Chapter 1: Linear Equations and Inequalities

1. Solve for y and check your answer.

$$\frac{2y-4}{5} = \frac{5y+13}{4} + \frac{y}{2}$$

2. Given the equation $ax - c = bx + d$

   (a) Solve for x.

   (b) Solve for $-x$.

3. Solve for s. $\frac{D}{r} + \frac{D}{r+s} = t$

4. (a) Which expressions are equivalent to $-\frac{x+7}{y}$?

   a. $\frac{x+7}{-y}$ b. $\frac{-x-7}{-y}$ c. $\frac{-x-7}{y}$

   (b) Which expressions are equivalent to $-\frac{3w}{-x-y}$?

   a. $\frac{3w}{x+y}$ b. $\frac{-3w}{-x-y}$ c. $-\frac{-3w}{x+y}$

5. Solve the compound inequality. Write the answer in interval notation.

   To validate the answer, plug into the inequality both endpoints and any interior point of the interval.

$$-4 \le \frac{6-2x}{5} < 2$$

6. Solve the absolute value equation. $|3y+1| = |2y-7|$
   Check both answers.

Chapter 2: Linear Equations, and Functions

7. Find the slope of the line passing through the points $(-4,-5)$ and $(8,31)$.

8. Given the equation $7x - 2y = 4$, find the slope and y-intercept. Then graph the equation.

9. Write an equation of the line that contains the point $(-2,5)$ and is perpendicular to a line with slope $\frac{1}{2}$. Write the answer in slope-intercept form or standard form. Then graph the equation and label both intercepts with their coordinates.

10. Write an equation of the line perpendicular to the line $2x + y - 4$ and passing through the point $(2,6)$. Write the answer in slope-intercept form.

11. Find the domain: $f(x) = \sqrt{2x-1}$.

Chapter 3: Systems of Linear Equations

12. Solve the system by using the addition method. Then write the solution as an ordered pair and check your answer.

$$2x + 5y = 9$$
$$4x - 7y = -16$$

13. Solve the system by graphing.

$$x + 3y = 6$$
$$2x + y = 7$$

14. Solve the system.

$$4x - 2y = 7$$
$$-3x + 5y = 0$$

15. Solve the system.

$$x = 1 - 2y$$
$$7y = 3x - 16$$

Chapter 4: Polynomials

16. Perform the indicated operations and simplify.
    (a) $4x(3x^2 - 7) - (2x^2 + 9)(6x - 1)$.
    (b) $(2x - 3)(x - 4) - (x - 5)^2$.

17. Multiply and express the result in scientific notation and in standard notation.

$$\frac{\left(3.1\times10^{-6}\right)\left(9\times10^{7}\right)}{1.2\times10^{3}}.$$

18. Simplify and write the answer with positive exponents only.

$$\left(\frac{-6a^2b^{-3}}{5a^{-1}b}\right)^{-2}$$

19. Simplify the expression, and write the answer with positive exponents only.

$$\frac{(3^{-1}xy^{-3})^{-3}(x^{-4}y)}{(x^0y^5)^{-1}}$$

20. Factor. $17y + 3y^2 - 28$

21. Factor completely. $8a^2 - 50$

22. (a) Factor completely. $15x^2 - 31x - 24$
    (b) Find the x-intercepts of the function $g(x) = 15x^2 - 31x - 24$.
    (c) What is the y-intercept?

23. Solve for x: $35x^2 + 16x = 3$.

Chapter 5: Rational Expressions and Rational Equations

24. Multiply these two rational expressions and simplify the result.

$$\left[-\left(\frac{2-3x+x^2}{2-x}\right)\right]\left[\frac{3-x}{9-x^2}\right]$$

25. Add these two rational expressions and simplify the result.

$$\frac{-10}{z^2-6z+5}+\frac{15}{z^2-4z-5}$$

26. Solve the equation. Be sure to check all answers.

$$\frac{x}{x+6}=\frac{72}{x^2-36}+4$$

27. Solve for x.

$$\frac{4}{x-2}=\frac{5}{x+10}+1.$$

28. Solve for x.

$$\frac{9}{x-4}+\frac{3}{x+2}=\frac{6}{x^2-2x-8}.$$

Chapter 6: Radicals

29. (a) Which of the following expressions are negative? Which are equal to each other?

$$(-8)^2, \quad -8^2, \quad \sqrt{(-64)^2}$$

(b) Which of the following expressions are negative? Which are equal to each other?

$$\sqrt[3]{-8}, \quad -\sqrt[3]{8}, \quad -\sqrt[6]{64}$$

30. Find the domain. $f(x) = \sqrt{2x-1}$.

31. (a) Evaluate the function values $f(-3)$, $f(0)$, $f(5)$, and $f(12)$ for $f(x) = \sqrt{x+4}$. Is it possible to evaluate $f(-5)$ over the real numbers?

(b) Write the domain of f in interval notation. State the exact coordinates of the endpoint of the graph of this equation.

(c) Plot these five points and use them to draw a rough sketch of the graph.

(d) For which x is $f(x) = 11$?

32. Simplify the expressions $(-8)^{-2/3}$ and $(-\frac{1}{3}) \cdot (-\frac{1}{3})^{-1/3}$

33. Simplify the expression $\sqrt{16^{-3/4} \cdot 16^{5/4} \cdot 16^{1/2}}$.

34. Simplify the expression $\sqrt[3]{16^{3/4} \cdot 8^{3/4}}$.

35. Add or subtract as indicated, simplifying as much as possible.

$8\sqrt{3} - 2\sqrt{27} + \sqrt{75}$.

36. Rationalize the denominator of each expression. $\dfrac{8}{\sqrt[3]{2}}$, $\dfrac{4}{\sqrt[3]{4}}$, $\dfrac{18}{\sqrt[3]{6}}$

37. Simplify the expression: $\dfrac{(\sqrt{x}+\sqrt{y})\,(\sqrt{x}-\sqrt{y})}{x^2-y^2}$.

38. Find an equivalent expression that is in the most simplified radical form.

$$\sqrt{\frac{8}{\sqrt[3]{2}}}.$$

39. Consider the expression: $\dfrac{\sqrt{2x}-\sqrt{3}}{\sqrt{2x}+\sqrt{3}} + \dfrac{\sqrt{2x}+\sqrt{3}}{\sqrt{2x}-\sqrt{3}}$.

(a) Evaluate the expression for $x = 6$ with answer a rational number (a fraction).

(b) Simplify the expression when x is not specified. The answer will be a rational expression in x.

40. Solve. $\sqrt{8x+5} = \sqrt{2x} + 2$

Chapter 7: Quadratic Equations and Functions

41. Find the value of n so that the expression is a perfect square trinomial. Then factor the trinomial $x^2 + x + n$, using the value of n you found.

42. Simplify the expression. $\dfrac{18 + \sqrt{180}}{3}$

43. Solve the equations $2x^2 - 3x = 5$ and $32z^2 - 20z = 3$.

44. An object is fired straight up into the air from an initial height of 384 ft with an initial velocity of 160 ft/sec. The height in feet is give by

$$h(t) = -16t^2 + 160t + 384$$

where t is the time in seconds after the launch.

(a) i. Find the height of the object after 3 sec.
ii. Find the height of the object after 7 sec.
iii. Find the time required for the object to hit the ground.

(b) i. Find the time when the object attains its maximum height.
ii. Find that maximum height.

(c) Was the object ever again at a height of 384 ft and, if so, when?

45. Given the equation

$$x^2 - 14x + 1 = 0.$$

(a) Solve by completing the square and using the square root property, leaving the answers in simplified radical form (no calculators permitted):

(b) Solve by using the quadratic formula.

(c) Which method did you find easier to use?

46. For the function defined by $g(x) = 4x^2 + 16x + 19$, find the vertex by using two methods.

(a) Complete the square to write $g(x)$ in the form $g(x) = a(x - h)^2 + k$. Identify the vertex.

(b) Use the vertex formula to find the vertex.

(c) Plot the vertex and the y-intercept. Then draw the axis of symmetry and plot the reflected point of the y-intercept.
Note: the axis of symmetry will be a vertical line through the vertex and the reflected point is obtained by reflecting the y-intercept across the axis of symmetry.

Use these to graph the function.

47. Given $f(x) = x^2 + 6x + 5$.

(a) Determine the x-intercepts.

(b) Determine the y-intercept.

(c) Determine the vertex of the parabola.

(d) Find the point of which the value of y is the same as at the y-intercept.
(This point is called the symmeteric point of the y-intercept.)

Word Problems.

48. The sum of three consecutive odd integers is 41 less than four times the largest. Find the integers.

    Check your answer.

49. The square of an odd integer less the next greater odd integer equals 54.

    Find the value of the odd integer being squared.

50. A grocer sells one grade of ground beef at \$3.95 per pound and another grade at \$4.20 per pound. How many pounds of each grade should be mixed to produce 20 pounds selling for \$4.15 per pound?

51. A driver makes the trip from Boston to Hartford at an average speed of 40 mph and covers the same distance back at an averge speed of 50 mph. The total time for both trips was four-and-a-half hours.

    (a) How long did the trip to Hartford take?

    (b) What is the distance from Boston to Hartford?

    Check your answers. Use of a calculator is suggested.

52. Shawna has money distributed between two accounts: an account that earns 5% simple interest and an account that earns 3.5% simple interest. She has \$100 less invested at 3.5% than at 5%. If after 1 year her total interest is \$81.50, how much did she invest at 5%?

    Check your answers on a calculator.

53. Nebuchadnezzar and Chrysanthemum bought snacks for an office break. Nebuchadnezzar spent \$18.75 on 12 donuts and 5 coffees. Chrysanthemum spent \$11.10 on 7 donuts and 3 coffees. What is the cost of 1 donut and what is the cost of 1 coffee?

54. A driver travels 30 miles to the gym at a constant speed and travels the same distance home at a constant speed. His speed on the return trip was sixteen miles per hour faster, and the total time for both trips was 2 hrs.

    Find his speed on the way to the gym.

    Check your work.

55. The length of a rectangle is 3 feet longer than a side of a square while the width of the rectangle is 5 feet shorter than a side of that square. The area of the square is 45 square feet greater than the area of the rectangle.

    Find the length of a side of that square.

Answers are on the following pages.

Answers.

1. $y = -3$ and $\dfrac{2(-3)-4}{5} = \dfrac{5(-3)+13}{4} + \dfrac{-3}{2}$ because $-2 = -1/2 - 3/2$.
2. $x = \dfrac{c+d}{a-b}$
3. $s = \dfrac{2Dr - tr^2}{tr - D}$.
4. (a) a. and c. (b) all of them
5. $-2 < x \leq 13$. When $x = -2$, $\frac{6-2(-2)}{5} = \frac{10}{5} = 2$; when $x = 13$, $\frac{6-2(13)}{5} = \frac{-20}{5} = -4$. Plug in $x = 0$ and get $-4 \leq 1.2 < 2$, which is true.
6. $x = -8$ or $x = 1.2$.
   $x = -8$: $|3(-8)+1| = |-23| = 23$ and $|2(-8)-7| = -16-7| = |-23| = 23$.
   $x = 1.2$: $|3(1.2)+1| = |3.6+1| = |4.6| = 4.6$ and $|2(1.2)-7| = |2.4-7| = |-4.6| = 4.6$.
7. $m = 3$.
8. $m = 7/2$, y-intercept is $(0, -2)$. Find another point such as $(2, 5)$ and connect these two points with a straight line (graph omitted).
9. $y = -2x + 1$ or $2x + y = 1$. x-intercept: $(\frac{1}{2}, 0)$, y-intercept: $(0, 1)$. Find a third point, say $(1, -1)$ and draw a straight line through these 3 points.
10. $y = \frac{1}{2}x + 5$.
11. $\left[\dfrac{1}{2}, \infty\right)$.
12. $(-\frac{1}{2}, 2)$ Check: $2(-\frac{1}{2})+5(2) = -1+10 = 9$ and $4(-\frac{1}{2})-7(2) = -2-14 = -16$.
13. $x = 3$ and $y = 1$.
    Draw the first line between $(0, 2)$ and $(6, 0)$.
    Then draw the second line through $(0, 7)$ and $(3\frac{1}{2}, 0)$.
    These two lines surely meet in the first quadrant. If carefully drawn, one can see that the point of intersection is located at about $(3, 1)$.
14. $\left\{\left(\dfrac{5}{2}, \dfrac{3}{2}\right)\right\}$.
15. $\{(3, -1)\}$.
16. (a) $2x^2 - 82x + 9$
    (b) $x^2 - x - 13$
17. 2.325×10^{-1} or 0.2325
18. $\dfrac{25b^8}{36a^6}$
19. $\dfrac{27y^{15}}{x^7}$
20. $(3y - 4)(y + 7)$
21. $2(2a - 5)(2a + 5)$
22. (a) $(3x - 8)(5x + 3)$ (b) $8/3$ and $-3/5$ (c) -24

23. $x = 1/7$ or $x = -3/5$.

24. $\dfrac{x-1}{x+3}$

25. $\dfrac{5}{z^2-1}$ or $\dfrac{5}{(z-1)(z+1)}$.

26. $x = 4$, giving $\frac{4}{10} = \frac{72}{-20} + 4$, then $0.4 = -3.6 + 4$, which checks. A potential answer of $x = 6$, a solution to the resulting quadratic, is inadmissible.

27. $x = 5$ or $x = -14$.

28. $x = 0$.

29. (a) only -8^2 is negative and the first and third expressions are equal.
    (b) All three are negative and equal.

30. $\left[\frac{1}{2}, \infty\right)$

31. (a) The four values are: 1, 2, 3, 4. $f(-5)$ cannot be evaluated as a real number.
    (b) $[-4, \infty)$. The endpoint is at $(-4, 0)$,
    (c) Graph omitted. It starts at $(-4, 0)$, passes through $(0, 2)$ and continues in the first quadrant. The shape is a half-parabola with the axis of symmetry the x-axis.
    (d) $x = 117$.

32. $1/4$ and $\sqrt[3]{3}/3$.

33. 4.

34. $2\sqrt[4]{8}$.

35. $7\sqrt{3}$.

36. $4\sqrt[3]{4}$, $2\sqrt[3]{2}$, $3\sqrt[3]{36}$

37. $\dfrac{1}{x+y}$.

38. $2\sqrt[3]{2}$.

39. (a) 10/3 (b) $\frac{4x+6}{2x-3}$

40. $\left\{\frac{1}{18}, \frac{1}{2}\right\}$

41. $n = \frac{1}{4}$ and $x^2 + x + \frac{1}{4}$ factors as $(x + \frac{1}{2})^2$.

42. $6 + 2\sqrt{5}$.

43. $x = -1$ or $x = 5/2$; and $x = -1/8$ or $x = 3/4$.

44. (a) i. 720 ft
        ii. 720 ft
        iii. 12 sec
    (b) i. 5 sec
        ii. 784 ft
    (c) Yes, at time 10 sec.

45. (a) $7 \pm 4\sqrt{3}$

$x^2 - 14x = -1$, $x^2 - 14x + 49 = -1 + 49$, $(x-7)^2 = 48$, $x - 7 = \pm\sqrt{48}$, $x = 7 \pm 4\sqrt{3}$.

(b) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{14 \pm \sqrt{14^2 - 4(1)}}{2} = \dfrac{14 \pm \sqrt{192}}{2} = \dfrac{14 \pm 8\sqrt{3}}{2}$ $= 7 \pm 4\sqrt{3}$.

(c) Completing the square was far easier. Using the quadratic formula in this problem makes for more frequent errors.

46. (a) $4(x^2 + 4x) + 19 = 4(x^2 + 4x + 4 - 4) + 19 = 4(x+2)^2 + 4(-4) + 19 = 4(x+2)^2 - 16 + 19 = 4(x+2)^2 + 3$, so the vertex is at $(-2, 3)$.

(b) $\left(\dfrac{-(16)}{2(4)}, \dfrac{4(4)(19) - 16^2}{4(4)}\right) = \left(-2, \dfrac{48}{16}\right) = (-2, 3)$.

(c) The y-intercept is at $(0, 19)$; the axis of symmetry is the vertical line $x = -2$. The reflected point ends up at $(-4, 19)$. The parabola opens up and is in the first and second quadrants. Graph omitted.

47. (a) $(-1, 0)$, $(-5, 0)$

(b) $(0, 5)$

(c) $(-3, -4)$

(d) $(-6, 5)$

48. The integers are 31, 33, and 35.

Check: $(31 + 33 + 35) = 4(35) - 41$, which is $99 = 99$. It checks.

49. The odd integer being squared must be -7. Minus seven squared is $+49$, then subtract -5 to get 54. (Eight solves the equation but it is not odd.)

50. Mix 4 pounds of the \$3.95 grade and 16 pound of the \$4.20 grade.

51. (a) Two-and-a-half hours. (b) 100 miles.

52. 1,000. Check: $0.05(1000) + 0.035(900) = 50 + 31.50 = 81.50$.

53. A donut costs \$0.75, while a coffee costs \$1.95.

54. 24 miles per hour.

Check: $\frac{30}{24} + \frac{30}{24+16} = 1.25 + \frac{30}{40} = 1.25 + 0.75 = 2$. It checks.

55. The length of a side of that square is 15 feet.