

# Sample Final Examination

Math 115 College Algebra  
(Exam on Dec. 20, 2019)

## Chapter 1: Linear Equations and Inequalities

1. Solve for  $y$  and check your answer.

$$\frac{2y - 4}{5} = \frac{5y + 13}{4} + \frac{y}{2}$$

2. Given the equation  $ax - c = bx + d$

- (a) Solve for  $x$ .
- (b) Solve for  $-x$ .

3. Solve the compound inequality. Write the answer in interval notation.

To validate the answer, plug into the inequality both endpoints and any interior point of the interval.

$$-4 \leq \frac{6 - 2x}{5} < 2$$

4. Solve the absolute value equation.  $|3y + 1| = |2y - 7|$   
Check both answers.

## Chapter 2: Linear Equations, Functions

5. Find the slope of the line passing through the points  $(-4, -5)$  and  $(8, 31)$ .
6. Find an equation for the line that passes through the points  $(3, -8)$  and  $(2, -4)$ . Write the answer in slope-intercept form.
7. Decide whether the straight lines determined by the equations  $3y = 5x + 6$  and  $5y + 3x = 6$  are parallel, perpendicular, or neither.
8. Let  $m(x) = \sqrt{2x + 1}$ .
  - (a) Find the domain and range of  $m$ .
  - (b) Find the coordinates of both intercepts and one point each in Quadrants I and II.
  - (c) Plot these four points and use the result to draw a rough graph.

## Chapter 3: Systems of Linear Equations

9. Solve the system by graphing.

$$x + 3y = 6$$

$$2x + y = 7$$

## Chapter 4: Polynomials

10. Multiply out.  $(7x - 3)(3x - 7)$
11. Subtract  $3x^2 - 5x - 5$  from  $x^2 - 6x + 4$ .  
Be careful. *Which is the expression being subtracted? From what it is being subtracted?*
12. Simplify.  $(-4a^{-4})^{-4} \cdot (4a^4)^4$
13. Factor.  $17y + 3y^2 - 28$
14. Factor completely.  $15x^2 - 31x - 24$

## Chapter 5: Rational Expressions and Rational Equations

15. Simplify.  $\left(\frac{1}{x} - y\right)\left(x + \frac{1}{y}\right)$
16. Solve for  $x$ .  
$$\frac{4}{x - 2} = \frac{5}{x + 10} + 1.$$

## Chapter 6: Radicals

17. Simplify.  $\left(\frac{81^{1/4} - 81^{3/4}}{81^{1/2}}\right)(16^{-3/4})$ .
18. Solve.
  - (a)  $\sqrt{x + 10} = 2 - x$ .
  - (b)  $\sqrt{x + 10} = x - 2$ .

## Chapter 7: Quadratic Equations and Functions

19. Solve for  $x$ . (Suggestion: for each part choose the easiest method.)
  - (a)  $x^2 - 6x + 5 = 0$ .
  - (b)  $x^2 - 6x - 5 = 0$ .
20. For the function defined by  $g(x) = 4x^2 + 16x + 19$ , find the vertex by using two methods.
  - (a) Complete the square to write  $g(x)$  in the form  $g(x) = a(x - h)^2 + k$ . Identify the vertex.
  - (b) Use the vertex formula to find the vertex.
  - (c) Plot the vertex and the  $y$ -intercept. Then draw the axis of symmetry and plot the reflected point of the  $y$ -intercept.  
Note: the axis of symmetry will be a vertical line through the vertex and the reflected point is obtained by reflecting the  $y$ -intercept across the axis of symmetry.  
Use these to graph the function.

## Word Problems.

21. Strykover Premium Slivovitz is 72% alcohol (144 proof). In order to bring it to a marketable 40% alcohol, a portion of the contents of a one-liter bottle is removed and replaced with a neutral grain spirit with no alcohol. What fraction of each liter should be removed and replaced to get a 40% blend?
22. A right triangle has a hypotenuse and two legs, one longer than the other. The hypotenuse is one unit longer than the leg of greater length. The shorter leg is one unit less than half the length of the longer leg. Find the length of the longer leg.
23. Nebuchadnezzar and Chrysanthemum bought snacks for an office break. Nebuchadnezzar spent \$18.75 on 12 donuts and 5 coffees. Chrysanthemum spent \$11.10 on 7 donuts and 3 coffees. What is the cost of 1 donut and what is the cost of 1 coffee?
24. A driver travels 225 miles to New York City at a constant speed and travels the same distance home at a constant speed. Because of heavy holiday traffic, his speed on the return trip was thirty miles per hour slower, and the total time for both trips was 8 hrs. Find his speed on the way to New York City. (Here, using the quadratic formula on a calculator is best.) Check your work.

Answers are on the following pages.

## Answers.

1.  $y = -3$  and  $\frac{2(-3) - 4}{5} = \frac{5(-3) + 13}{4} + \frac{-3}{2}$  because  $-2 = -1/2 - 3/2$ .
2.  $x = \frac{c+d}{a-b}$  and  $-x = \frac{c+d}{b-a}$ .
3.  $-2 < x \leq 13$ . When  $x = -2$ ,  $\frac{6-2(-2)}{5} = \frac{10}{5} = 2$ ; when  $x = 13$ ,  $\frac{6-2(13)}{5} = \frac{-20}{5} = -4$ . Plug in  $x = 0$  and get  $-4 \leq 1.2 < 2$ , which is true.
4.  $x = -8$  or  $x = 1.2$ .  
 $x = -8$ :  $|3(-8) + 1| = |-23| = 23$  and  $|2(-8) - 7| = |-16 - 7| = |-23| = 23$ .  
 $x = 1.2$ :  $|3(1.2) + 1| = |3.6 + 1| = |4.6| = 4.6$  and  $|2(1.2) - 7| = |2.4 - 7| = |-4.6| = 4.6$ .
5.  $m = 3$ .
6.  $y = -4x + 4$ .
7. They are perpendicular.
8. (a) The domain is  $\left[-\frac{1}{2}, \infty\right)$  and the range is  $[0, \infty)$ .  
 (b) The intercepts are  $(-\frac{1}{2}, 0)$  and  $(0, 1)$ .  
 A point in QI is  $(4, 3)$ ; a point in QII is  $(-\frac{4}{9}, \frac{1}{3})$ .  
 (c) Graph not shown.
9.  $x = 3$  and  $y = 1$ .  
 Draw the first line between  $(0, 2)$  and  $(6, 0)$ .  
 Then draw the second line through  $(0, 7)$  and  $(3\frac{1}{2}, 0)$ .  
 These two lines surely meet in the first quadrant. If carefully drawn, one can see that the point of intersection is located at about  $(3, 1)$ .
10.  $21x^2 - 58x + 21$
11.  $-x^2 - x + 9$
12.  $a^{32}$
13.  $(3y - 4)(y + 7)$
14.  $(3x - 8)(5x + 3)$
15. Either  $\frac{1}{xy} - xy$  or  $\frac{1 - x^2y^2}{xy}$ .
16.  $x = 5$  or  $x = -14$ .
17.  $-1/3$ .
18. (a)  $-1$ . (b)  $6$ .
19. (a) Factoring:  $x = 1$  or  $x = 5$ .  
 (b) Completing the square:  $x = 3 \pm \sqrt{14}$ .
20. (a)  $4(x^2 + 4x) + 19 = 4(x^2 + 4x + 4 - 4) + 19 = 4(x + 2)^2 + 4(-4) + 19 = 4(x + 2)^2 - 16 + 19 = 4(x + 2)^2 + 3$ , so the vertex is at  $(-2, 3)$ .  
 (b)  $\left(\frac{-(-16)}{2(4)}, \frac{4(4)(19) - 16^2}{4(4)}\right) = \left(-2, \frac{48}{16}\right) = (-2, 3)$ .  
 (c) The  $y$ -intercept is at  $(0, 19)$ ; the axis of symmetry is the vertical line  $x = -2$ . The reflected point ends up at  $(-4, 19)$ . The parabola opens up and is in the first and second quadrants. Graph omitted.

21. Four-ninths of a liter must be removed and replaced with neutral grain spirits.
22. 12 units.
23. A donut costs \$0.75, while a coffee costs \$1.95.
24. 75 miles per hour.

Check:  $\frac{225}{75} + \frac{225}{75-30} = 3 + \frac{225}{45} = 3 + 5 = 8$ . It checks.