Problems: chapters 13, 14, and 15

Math 125 *Kovitz* Fall 2020

1. A deck of cards is shuffled and placed face down on a table.

   Find the chance that the top three cards:

   (a) are all black.

   (b) are all red.

   (c) are not all black.

   (d) are not all red.

   (e) contain at least one card of each color.

2. Five cards are drawn from a well-shuffled deck.

   Find the chance that the first card drawn is red, the second card drawn is black, the third card drawn is red, the fourth card drawn is black, and the fifth card drawn is red, if the draws are made:

   (a) with replacement.

   (b) without replacement.

3. Find the chance as an exact fraction that, in a well-shuffled deck, either the top card is a club or the bottom card is red. (This is a challenge problem.)

4. Find the chance as an exact fraction that, in a well-shuffled deck, either the top card is a club or the bottom card is red, but not both. (This is a challenge problem.)

5. Four draws are made with replacement from a well-shuffled deck. Find the chance that

   (a) all of the cards are red.

   (b) none of the cards are red.

   (c) not all of the cards are red.

   (d) one of the four cards is red and the other three are black.

6. Same as the previous problem, except the draws are made without replacement.

Answers follow.

Answers.

1. (a) 2/17. It is $\frac{26}{52} \times \frac{25}{51} \times \frac{24}{50}$, with cancellation.
   (b) 2/17
   (c) 15/17. It's the opposite of all black, so its chance is $1 - 2/17 = 15/17$.
   (d) 15/17. It's the opposite of all red, so its chance is $1 - 2/17 = 15/17$.
   (e) 13/17.
   First find the chance of getting "all red or all black." The outcomes are mutually exclusive, so add their chances to get $2/17 + 2/17 = 4/17$.
   What we are looking for in this problem, at least one of each color, is the opposite of "all red or all black." Its chance is $1 - 4/17 = 13/17$.

2. (a) 1/32
   (b) 325/9996. It is $\frac{1}{2} \times \frac{26}{51} \times \frac{1}{2} \times \frac{25}{49} \times \frac{1}{2}$, with cancellation.

3. 127/204

   Three ways to answer this.

   (A) P(club)+P(red card)−P(both)= $1/4 + 1/2 - 13/102 = 127/204$.

   (B) P(club on top, or spade on top and red on bottom, or red on top and red on bottom)= $1/4 + 1/4(26/51) + 1/2(25/51) = 51/204 + 26/204 + 50/204 = (51 + 26 + 50)/204 = 127/204$.

   (C) P(red on bottom, or spade on bottom and club on top, or club on bottom and club on top)= $1/2 + 1/4(13/51) + 1/4(12/51) = (102 + 13 + 12)/204 = 127/204$.

4. 101/204

   Three ways to answer this.

   (A) Subtact the chance of getting both from 3(A)'s answer: $127/204 - 13/103 = 101/204$.

   (B) P(club on top and no red on bottom, or spade on top and red on bottom, or red on top and red on bottom)= $1/4(25/51) + 1/4(26/51) + 1/2(25/51) = 25/204 + 26/204 + 50/204 = (25 + 26 + 50)/204 = 101/204$.

   (C) P(red on bottom and non-club on top, or spade on bottom and club on top, or club on bottom and club on top)= $1/2(38/51)+1/4(13/51)+1/4(12/51) = 76/204+13/204+12/204 = 101/204$.

5. (a) 1/16.
   (b) 1/16. That's just all black, which has the same chance.
   (c) 15/16. It's the opposite of all red so $1 - 1/16 = 15/16$.
   (d) 1/4. Use the binomial formula with $n = 4$, $k = 1$, $p = 1/2$, and $1 - p = 1/2$.

6. (a) 46/833, or about 5.5%. It's (26/52)(25/51)(24/50)(23/49), after cancelling.
   (b) 46/833 or about 5.5%. That's just all black, which has the same chance.
   (c) 787/833. It's the opposite of all red so $1 - 46/833 = 787/833$, or about 94.5%.
   (d) 208/833. It is (26/52)(26/51)(25/50)(24/49), times 4 for the 4 possible orders. After cancellation, it ends up as 208/833.
   (Note: in each order the 4 factors in the numerator will be the same, just possibly in different order.)

   Coincidentally, the answer to part (d), when rounded to the nearest 1/10 of one percent, is 25.0%, just about the same as it was in part 5(d),without replacement.