

Model for Measurement Error Practice

1. The National Bureau of Standards is about to weigh a one-kilogram check weight 100 times, and take the average of the measurements. They are willing to assume the Gauss model, with no bias, and on the basis of past experience they estimate the SD of the error box to be 50 micrograms.
 - (a) The average of all 100 measurements is likely to be off the exact weight by _____ or so.
 - (b) The SD of all 100 measurements is likely to be around _____.
 - (c) Estimate the probability that the average of all 100 measurements will be within 10 micrograms of the exact weight.
2. Twenty-five measurements are made on the speed of light. These average out to 300,007 and the SD is 10, the units being kilometers per second. Fill in the blanks in part (a), then say whether each of (b–f) is true or false; explain your answers briefly. (You may assume the Gauss model, with no bias.)
 - (a) The speed of light is estimated as _____; this estimate is likely to be off by _____ or so.
 - (b) The average of all 25 measurements is off 300,007 by 2 or so.
 - (c) Each measurement is off 300,007 by 10 or so.
 - (d) A 95%-confidence interval for the speed of light is $300,007 \pm 4$.
 - (e) A 95%-confidence interval for the average of the 25 measurements is $300,007 \pm 4$.
 - (f) About 95% of the readings were in the range $300,007 \pm 4$ kilometers per second.
 - (g) If a 26th measurement were made, there is a 95% chance that it would be off the exact value for the speed of light by less than 4.
3. The speed of light was measured 2,500 times. The average was 299,774 kilometers per second, and the SD was 14 kilometers per second. Assume the Gauss model, with no bias. Find a 95%-confidence interval for the speed of light.
4. A machine makes sticks of butter whose average weight is 4.0 ounces. The SD of the weights is 0.05 ounces. There is no trend or pattern to the data. There are 4 sticks to a package.
 - (a) A package weighs _____, give or take _____ or so.
 - (b) A store buys 100 packages. Estimate the chance that they get 100 pounds of butter, to within 2 ounces.
5. True or false and explain: “If the data don’t follow the normal curve, you can’t use the curve to get confidence levels.”

Answers follow on the next page.

Answers.

1. The SD of the error box is estimated as 50 micrograms.
 - (a) 5 micrograms—the SE for the average.
 - (b) 50 micrograms—the estimated SD of the error box.
 - (c) 95%—two SEs.
2.
 - (a) 300,007 (the average); 2 (the SE for the average).
 - (b) False: the average is 300,007 exactly.
 - (c) True: each number on a list is off the average of the list by an SD or so.
 - (d) True: the interval is “average \pm 2SEs.”
 - (e) False: the average of the 25 measurements is 300,007 exactly.
 - (f) False: this mixes up SD and SE.
 - (g) False: 2 is the SE, not the SD.
3. $299,774 \pm 0.6$ km/sec.
4.
 - (a) Model: the weights of the sticks are drawn at random from a box. The average of the box is 4 ounces, and the SD is 0.05 ounces. The weight of a package is like the sum of 4 draws. The expected value is $4 \times 4 = 16.0$ ounces. The SE for the sum is $\sqrt{4} \times 0.05 = 0.1$ ounces. The answer: 16 ounces, 0.1 ounces.
 - (b) The total weight of 100 packages is like the sum of 400 draws from the box. The expected value is
$$400 \times 4 = 1600 \text{ ounces} = 100 \text{ pounds.}$$
The SE for the sum is
$$\sqrt{400} \times 0.05 = 1 \text{ ounce.}$$
The total weight will be 100 pounds, give or take 1 ounce or so. The range “100 pounds \pm 2 ounces” is “expected value \pm 2 SE,” so the chance is about 95%.
5. False. You use the curve on the probability histogram for the average, not the histogram for the data (pages 418–419).