Practice in Independence and Mutual Exclusivity

For Problems 1–5:

Each of two boxes contains ten tickets, marked 1 through 10. One ticket is drawn at random from each box.

For each of Problems 1–5:

- Find P(A) and P(B); are A and B independent? mutually exclusive?
- Find P(both A and B). Would multiplying the unconditional probabilities, P(A) and P(B), give the correct answer, or are conditional probabilities required? Why or why not?
- Find P(at least one of A or B)=P(A or B). Would adding the individual probabilities of A and B give the correct answer? Why or why not?
- Find P(B|A) and decide if it is equal to P(B). Based on this decision, are A and B independent?
- 1. A: both tickets drawn have odd numbers.

B: the sum of the numbers on the two tickets drawn is an odd number.

- 2. A: the number on the first ticket drawn is the 4.B: the sum of the numbers on the two tickets drawn is an odd number.
- 3. A: both tickets drawn have odd numbers.B: the sum of the numbers on the two tickets drawn is an even number.
- 4. A: the number on the first ticket drawn is an even number.B: the number on the first ticket drawn is greater than 7.
- 5. A: the number on the first ticket drawn is an even number.B: the sum of the numbers on the two tickets drawn is 21.

For Problems 6–7:

Three tickets are drawn from a box that contains ten tickets, marked 1 through 10.

For each of Problems 6-7:

Find the chance that:

- All the tickets drawn have numbers greater than 3,
- At least one ticket drawn has a number greater than 3,
- All the tickets drawn have numbers that are 3 or less,
- All the tickets drawn have numbers that are 2 or less,
- Not all the numbers on the tickets drawn are greater than 3,
- At least one of the three tickets drawn has the number 10 on it,

when the draws are made

- 6. with replacement.
- 7. without replacement.

Answers are on the reverse side of the page.

Answers.

- 1. 1/4, 1/2, not independent, mutually exclusive.
 0, conditional probabilities needed, they are not independent.
 3/4, yes, they are mutually exclusive.
 0, not equal, so not independent.
- 1/10, 1/2, independent, not mutually exclusive.
 1/20, conditional probabilities not needed, they are independent.
 11/20, no, they are not mutually exclusive.
 1/2, equal, so indepenent.
- 3. 1/4, 1/2, not independent, not mutually exclusive.
 1/4, conditional probabilities needed, they are not independent.
 1/2, no, they are not mutually exclusive.
 1, not equal, so not independent.
- 4. 1/2, 3/10, not independent, not mutually exclusive.
 - 1/5, conditional probabilities needed, they are not independent.
 - 3/5, no, they are not mutually exclusive.
 - 2/5, not equal, so not independent.
- 5. 1/2, 0, independent, mutually exclusive.
 0, conditional probabilities not needed, they are independent.
 1/2, yes, they are mutually exclusive.
 - 0, equal, so independent.
- 6. 343/1000 or 34.3%

973/1000 or 97.3% 27/1000 or 0.27%

8/1000 = 1/125 or 0.08%

 $657/1000~{\rm or}~65.7\%$

 $271/1000~\mathrm{or}~27.1\%$

7. 21/72 or about 29.167%

119/120 or about 99.167%

1/120 or about 0.833%

0%

51/72 or about 70.833%

3/10 or 30% (Note that the *unconditional* chances must be added: $\frac{1}{10} + \frac{1}{9} + \frac{1}{8}$ is not the correct answer.)