The Key Probability Formulas: the case of "and" or "both"

The Chance of Both A and B Happening: P(A and B)

This chance can always be found using the Multiplication Rule.

First decide whether A and B are independent.

Remember: two things are *independent* if the chances for the second given the first are the same, no matter how the first one turns out. Otherwise, the two things are *dependent*.

(both/and) **Case I: A and B are Independent.**

The Chance that Both of Them Will Happen is

$$\text{P(both A and B)} = \text{P(A and B)} = \text{P(A)} \times \text{P(B)}$$

This may be stated verbally:

"If two events are independent, the chance that both will happen equals the product of their unconditional probabilities." This is a special case of the multiplication rule.

There are three important points here.

- Conditional probabilities are not necessary when the events are independent. So here: $\text{P(B|A)} = \text{P(B)}$.
- The chance being calculated is that both A *and* B will happen. The conjuction is '*and*'.
- Blindly multplying chances can make real trouble. Be sure to check for independence, or use conditional probabilities.

(both/and) **Case II: A and B are Dependent.**

The Chance that Both of Them Will Happen is

$$\text{P(both A and B)} = \text{P(A and B)} = \text{P(A)} \times \text{P(B|A)}$$

This may be stated verbally:

"The chance that two things will both happen equals the chance that the first will happen, multiplied by the chance that the second will happen given that the first has happened."

- When the two events are not independent, conditional probabilities will be needed.

The Key Probability Formulas: the case of "at least one" or "or"

The Chance of at Least One of A and B Happening: P(A or B)

First decide whether A and B are mutually exclusive.

Remember: two things are *mutually exclusive* when the occurrence of one prevents the occurrence of the other: one excludes the other.

(at least one/or) **Case I: A and B are Mutually Exclusive**

The Addition Rule will apply

To find the chance that at least one of two things will happen, check to see if they are mutually exclusive. If they are, add the chances.

The Chance that At Least One of Them Will Happen is

$$\text{P(at least one of A and B)} = \text{P(A or B)} = \text{P(A)} + \text{P(B)}$$

This may be stated verbally:

"If two events are mutually exclusive, the chance that either will happen equals the sum of their unconditional probablilites." This is called the "addition rule."

There are two important points here.

- When calculating the chance of B, do not assume that A has already happened: the formula does not call for the conditional probablility of B given A, but rather P(B), the unconditional chance of B happening. Be careful with this point.
- The chance being calculated is that either A *or* B will happen.

  There are several ways of stating this: 'either A or B', 'at least one of A or B', 'A or B or both', 'A and/or B', or just 'A or B'. The conjuction is always '*or*'.
- As long as they are all mutually exclusive, the addition rule may extended to three or more events.

(at least one/or) **Case II: A and B are Not Mutually Exclusive**

If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big.

Blindly adding the chances can give the wrong answer, by double-counting the chance that both things happen. With mutually exclusive events, there is no double-counting.

There are several methods to find the chance of either of two events when the events are not mutually exclusive. One is the method of the Chevalier de Méré, presented below as the "Procedure of de Méré when the things are not mutually exclusive." When applying that procedure, it is good to know that the opposite of 'A or B' is 'not A and not B,' or—as otherwise stated—the opposite of 'either A or B' is 'neither A nor B.' Also note that the opposite of 'at least one of A and B' is 'none of A or B.' For more than two events, the opposite of 'at least one of several events happens' is 'none of them happens' or 'all of them do not occur,' (not A and not B and not C, etc.).

Procedure of de Méré when the things are not mutually exclusive:

- The opposite thing is that none of those things will happen. This, the opposite thing, means that each of the original things did *not* happen. Its chance can now be calculated by repeated application of the multiplication rule. (With 2 things, only 1 application is needed.)
- The original chance that we desired—namely the chance that at least one of several things will happen—can now be found by subtracting the chance of its opposite from 100% (or from 1 if using fractions or decimals).

Finding the Chance that at Least one of A and B will happen

The Chance that At Least One of Them Will Happen is

$$\text{P(at least one of A and B)} = \text{P(A or B)} < \text{P(A)} + \text{P(B)}$$

The Addition Rule does not apply when A and B are not mutually exclusive.

Find the opposite event of 'at least one of A and B' (the opposite of 'A or B').

That will be 'not A and not B.'

Use the Multiplication Rule to find P(not A and not B).

Depending on whether A and B are independent or not, that chance will be:

P(not A) × P(not B)

or P(not A) × P(not B)|(not A).

Subtract that probability from 100% or from 1 to get the desired chance.

Formula for the de Méré Method

$$\text{P(at least one of A and B)} = \text{P(A or B)} = 1 - [\text{P(not A)} \times \text{P(not B)}]$$

(If 'not A' and 'not B' are dependent, replace P(not B) with P(not B|not A).)