**List of Formulas, Procedures and Boxes for Quiz 6, May 14**

(Quiz 6 will be on Wednesday, May 14, and covers material from Chapter 28.1 and 28.2.)

Math 125 *Kovitz* Spring 2025

# From Text

Example 1 on pages 524 to 529.

Box on page 527.

Section 28.2 on pages 530 and 531.

# Formulas

## Tests of Significance: The Null and the Alternative

To make a test of significance, the null hypothesis has to be formulated as a statement about a box model. Usually, the alternative does too.

- The *null hypothesis* says that an observed difference just reflects chance variation.
- The *alternative hypothesis* says that the observed difference is real.

The null hypothesis expresses the idea that an observed difference is due to chance. To make a test of signficance, the null hypothesis has to be set up as a box model for the data. The alternative hypothesis is another statement about the box; it says that the difference is real.

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## Testing Hypotheses: The Observed Significance Level

A test statistic is used to measure the difference between the data and what is expected on the null hypothesis.

$z$ says how many SEs away an observed value is from its expected value, where the expected value is calculated using the null hypothesis. The formula is

$$z = \frac{\text{observed value} - \text{expected value}}{\text{standard error}}.$$

The observed significance level—often called the $P$-value—is the chance of getting a test statistic as extreme as or more extreme than the observed one. The chance is computed on the basis that the null hypothesis is right. The smaller the chance is, the stronger the evidence against the null.

A small value of P indicates that an explanation saying that this is chance variation is unreasonable. We cannot accept the model stated in the null.

A large value of P could very well be due to chance variation and we accept the null as a reasonable model.

The $P$-value of a test is the chance of getting a big test statistic—assuming the null hypothesis to be right. $P$ is not the chance of the null hypothesis being right.

## Making a Test of Significance

Based on some available data, the investigator has to—

- translate the null hypothesis into a box model for the data;
- define a test statisic to measure the difference between the data and what is expected on the null hypothesis;
- compute the observed significance level $P$.

The choice of test statistic depends on the model and the hypothesis being considered.

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## Making the Decision

Many statisticians have a dividing line that indicates how small the observed significance level has to be before an investigator should reject the null hypothesis.

- If $P$ is less than 5%, the result is called *statistically significant* and the null hypothesis is rejected.
- If $P$ is greater than 5%, we accept the null and state that chance error is a reasoanable explanation for this result.

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## The Chi-Square Test ($\chi^2$-Test)

The $\chi^2$-test is used to check whether a box model for classification involving more than two categories is appropriate in view of certain observed data. This is an approximation to the actual probabilities and may be trusted when each expected frequency in the table is 5 or more.

The $\chi^2$-statistic is obtained by evaluating

$$\chi^2 \;=\; \text{sum of} \;\; \frac{(\text{observed frequency} - \text{expected frequency})^2}{\text{expected frequency}}$$

There is one term in the sum for each line in the table listing observed and expected frequencies.

This statistic measures the distance between observed and expected frequencies.

For the $\chi^2$-test, $P$ is approximately equal to the area to the right of the observed value for the $\chi^2$-statistic, under the $\chi^2$-curve with the appropriate number of degrees of freedom.

When the model is fully specified (no parameters to estimate),

$$\text{degrees of freedom} \;=\; \text{number of terms in } \chi^2 \;-\; \text{one.}$$

- The $\chi^2$-test says whether the data are like the result of drawing at random from a box whose contents are given.
- The $z$-test says whether the data are like the result of drawing at random from a box whose average is given.

The ingredients of the $\chi^2$-test are the basic data, chance model, frequency table, $\chi^2$-statistic, degrees of freedom, and the observed significance level.