## Solutions to Practice for Test 1, Math 125

(The test will be on March 14)

Math 125 Kovitz Spring 2024

On the test, show your work.

- 1. Answer: (B) No. The normal curve is a curve with long tails and decreasing density farther from the center. This histogram consists of six blocks with constant density.
- 2. Answer: (C) more. When you put the little people and the basketball players together, the spread goes up, because the two distributions are different.

Most members of each group are around 3 inches from the average of the group. The average of the two groups together is 59 inches and, since there are no members of the combined group within 8 inches of that number, the SD for the combined group is surely more than 8 inches, so it is more than the original 3 inches.

3. This problem is similar to Example 10 on pages 90 and 91.

First find the standard units for the 20th percentile.

The definition of the 20th percentile requires a height with 20% of the heights below it. Let's find the -z (it'll be negative) for that height. Below -z standard units will be 20% of the area. By symmetry, above z, will also be 20% of the area. So, between -z and z will be 100% - 20% - 20% = 60% of the area.

From the table, we see that  $-z \approx -0.85$ . The 20th percentile height in standard units (z) is -0.85.

Carefully review the box on page 79. A student has to be about 0.85 SDs below average to be at the 20th percentile of the height distribution. This is the key point of chapter 5.1: A negative value in standard units tells you how many SDs the height is below average. Translated back to inches, this height is below average by  $0.85 \times (3 \text{ inches}) = 2.55$  inches.

The 20th percentile of the height distribution is 69 - 2.55 = 66.45 inches.

Answer: (B) 66.45 inches

4. The average 
$$x = \frac{1+4+8+10+12}{5} = 7$$
.  
The SD of  $x = \sqrt{\frac{(1-7)^2 + (4-7)^2 + (8-7)^2 + (10-7)^2 + (12-7)^2}{5}} = \sqrt{\frac{36+9+1+9+25}{5}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$ .  
The average  $y = \frac{10+11+7+13+9}{5} = 10$ .  
The SD of  $y = \sqrt{\frac{(10-10)^2 + (11-10)^2 + (7-10)^2 + (13-10)^2 + (9-10)^2}{5}} = \sqrt{\frac{0+1+9+9+1}{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$ .

The averages are 7 and 10, respectively; the SDs are 4 and 2, respectively.

Put all 10 values into standard units using the formula  $\frac{\text{observation -average}}{\text{SD}}$ The first calculation for x is  $\frac{1-7}{4} = \frac{-6}{4} = -1.5$ . The others for x are:  $\frac{4-7}{4} = -0.75, \ \frac{8-7}{4} = 0.25, \ \frac{10-7}{4} = 0.75, \ \frac{12-7}{4} = 1.25.$ 

The calculations for the y's are:

$$\frac{10-10}{2} = 0, \ \frac{11-10}{2} = 0.5, \ \frac{7-10}{2} = -1.5, \ \frac{13-10}{2} = 1.5, \ \frac{9-10}{2} = -0.5.$$

The average product of the values in standard units is then

$$\frac{(-1.5 \times 0) + (-0.75 \times 0.5) + (0.25 \times -1.5) + (0.75 \times 1.5) + (1.25 \times -0.5)}{6}$$
$$= \frac{0 + (-0.375) + (-0.375) + 1.125 + (-0.625)}{5} = \frac{-0.25}{5} = -0.05.$$

Note: The alternate method on page 134 gives  $\frac{348/5 - (7)(10)}{(4)(2)} = \frac{69.6 - 70}{8} = \frac{-0.4}{8} = -0.05.$ 

Answer: (A) - 0.05

- 5. See the box on page 128.
  - (a) False. It's the opposite: As weight increases, education decreases.
  - (b) True, as education increases, weight decreases.
  - (c) True. See the first bullet in the box on page 143.
  - (d) False. The correlation will reamin the same. Read the last paragraph on page 141 continued on page 142. Change of scale is an instance of the third bullet on the box on page 143.
  - (e) True, as weight decreases, education increases.
  - (f) False. Association is not causation. See the box on page 150.
- 6. (a) Use the simple formula:

estimated average  $y = \text{average } y + [(x \text{ in standard units}) \times r \times (\text{the SD of } y)].$ 

Their age is  $\frac{65-50}{15} = \frac{15}{16} = 0.9375$  in standard units.

The formula yields:  $13.2 + [0.9375 \times (-0.20) \times 3] = 13.2 + (-0.5625) = 12.6375$  years of education.

Alternative Solution without the Formula: (See the top half of page 160 in the text.)

Their age of 65 years is 15 pounds above 50 years, the average. That is 15/16 = 0.9375 SDs above average. Those 65-year-old women should be r times 0.9375 SDs above average in aget. So they are  $-0.20 \times 0.9375 =$ 

-0.1875 SDs above average in education. That means 0.1875 SDs below average in educational level That's  $0.1875 \times 3$  years = 0.5625 years. So their average is around 13.2 - 0.5625 = 12.6375 years.

Answer: 12.6375 years.

- (b) Answer: (B) False. According to part (a), women her age averaged 12.6375 years of education.
- 7. Answer: (C) between the 10th and 50th. This is the regression effect (box on page 169).
- 8. Answer: (E) 24. See Figure 3 on page 183 and the box on page 182, and read the paragraph below that box. About 99.73% of the points on a scatter diagram fall inside the strip whose edges are parallel to the regression line, and three r.m.s. errors away (up or down).

Three r.m.s. errors is 3 times 8 points, 24 points.