List of Formulas for Math 125: Chapters 13, 14, and 15

Formulas

The Chance of the Opposite

The chance of something equals 100% minus the chance of the opposite thing.

P(A) = 100% − P(not A). Also, P(not A) = 100% − P(A).

In decimal or fraction form: P(A) = 1 − P(not A), or P(not A) = 1 − P(A).

The Multiplication Rule

The chance that two things will both happen equals the chance that the first will happen, multiplied by the chance that the second will happen given that the first has happened. **P(A and B) = P(A) × P(B|A)**.

Independence

Two things are *independent* if the chances for the second given the first are the same, no matter how the first one turns out. Otherwise, the two things are *dependent*.

When drawing at random with replacement, the draws are independent. Without replacement, the draws are dependent.

The Chance that Both of Two Things Will Happen

Special case when A and B are independent (meaning that P(B|A) = P(B)):

$$\mathrm{P(both\ A\ and\ B) = P(A\ and\ B) = P(A) \times P(B)}$$

This may be stated verbally:

"If two events are independent, the chance that both will happen equals the product of their unconditional probablilites. This is a special case of the multiplication rule."

There are three important points here.

- Conditional probabilities are not needed when the events are independent.
- The chance being calculated is that both A *and* B will happen. The conjuction is '*and*'.
- Blindly multplying chances can make real trouble. Be sure to check for independence, or use conditional probabilities.

Mutually Exclusive: Definition

Two things are *mutually exclusive* when the occurrence of one prevents the occurrence of the other: one excludes the other.

The Addition Rule

To find the chance that at least one of two things will happen, check to see if they are mutually exclusive. If they are, add the chances.

The Chance that At Least One of Two Things Will Happen

Special case when A and B are muutally exclusive (meaning that P(A and B) = 0);

$$\text{P(at least one of A and B)} = \text{P(A or B)} = \text{P(A)} + \text{P(B)}$$

This may be stated verbally:

"If two events are mutually exclusive, the chance that either will happen equals the sum of their unconditional probablilites." This is called the "addition rule."

There are two important points here.

- When calculating the chance of B, do not assume that A has already happened: the formula does not call for the conditional probablility of B given A, but rather P(B), the unconditional chance of B happening.
- The chance being calculated is that either A *or* B will happen.

  There are several ways of stating this: 'either A or B', 'at least one of A or B', 'A or B or both', 'A and/or B', or just 'A or B'. The conjuction is always '*or*'.

The Chance that at Least One of Several Things Will Happen

Special Case: When the things are mutually exclusive.

The chance that at least one of those things will happen reduces to the chance that exactly one of them will happen—the things being mutually exclusive means that there is no chance that more than one of them will happen. This chance can be calculated by the addition rule.

The Chance of A or B, when they are Not Mutually Exclusive

If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big.

Blindly adding the chances can give the wrong answer, by double-counting the chance that both things happen. With mutually exclusive events, there is no double-counting.

There are several methods to find the chance of either of two events when the events are not mutually exclusive. One is the method of the Chevalier de Méré, presented below as the "Procedure when the things are not mutually exclusive." When applying that procedure, it is good to know that the opposite of 'A or B' is 'not A and not B,' or—as otherwise stated—the opposite of 'either A or B' is 'neither A nor B.'

The Usefulness of the Opposite Event:

If the chance of an event is hard to find, try to find the chance of the opposite event. Then subtract from 100%. (See The Chance of the Opposite, above.) This is useful when the chance of the opposite is easier to compute.

The Chance that at Least One of Several Things Will Happen

(If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big. Blindly adding the chances can give the wrong answer, by double-counting chances where two or more of the things happen together. With mutually exclusive events, there is no double-counting.)

Procedure when two or more things are not mutually exclusive:

- The opposite thing is that none of those things will happen. This, the opposite thing, means that each of the original things did *not* happen. Its chance can now be calculated by repeated application of the multiplication rule. (With 2 things, only 1 application is needed.)
- The original chance that we desired—namely the chance that at least one of several things will happen—can now be found by subtracting the chance of its opposite from 100%.

Summary of the de Méré procedure for "at least one":

A) It might be easier to find the chance of the opposite event, especially when the events A, B, C, etc. are not mutually exclusive.

B) The opposite of "at least one" (same as A or B or C) is "none."

C) Find the chance of "none" (same as not A and not B and not C) using the multiplication rule.

D) This will be: P(none) = P(not A) $\times$ P(not B) $\times$ P(not C).

E) Subtract that result from 1 (or 100%).

The Binomial Formula

The chance that an event will occur exactly k times out of n is given by the binomial formula

$$\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

In this formula, n is the number of trials, k is the number of times the event is to occur, and p is the probability that the event will occur on any particular trial. The assumptions:

- The value of n must be fixed in advance.
- p must be the same from trial to trial.
- The trials must be independent.

The first factor in the binomial formula is the binomial coefficient.