List of Formulas, Procedures and Boxes for Test 2, April 14

(Test 3 will be on Monday, April 14, and covers material from Chapters 13, 14, 15, 17, and 18.)

Math 125 *Kovitz* Fall 2024

From Text

Second box on page 223.

Box on page 225.

Boxes on pages 229, 230, 231, 232, and 234.

Boxes on pages 241 (top box only), 242, and 250.

Discussion in 14.3 pages 243 and 244, up to just before Example 6.

Summary on page 254: points 2 and 3.

Box on page 259.

Boxes on page 289 and 291, and the top box on page 292.

Box on page 298.

Box on page 301.

Section 18.4: up to the solution of Example 1(c) on top third of page 318.

Example 1(a) on page 317.

Boxes on pages 325 and 326.

Formulas

Conversion to Standard Units

An observed sum is converted to standard units by seeing how many SEs it is above or below the expected value.

The formula is: $$\text{standard units} = \frac{\text{observed sum} - \text{expected value for the sum}}{\text{SE for sum}}.$$

The Normal Approximation for Data

If a histogram follows the normal curve, approximate areas may be found by converting the endpoints to standard units and finding the appropriate areas under the normal curve by using the Table.

The Chance of the Opposite

The chance of something equals 100% minus the chance of the opposite thing.

The Multiplication Rule

The chance that two things will both happen equals the chance that the first will happen, multiplied by the chance that the second will happen given that the first has happened. $\mathbf{P(A \text{ and } B) = P(A) \times P(B|A)}$.

Independence

Two things are *independent* if the chances for the second given the first are the same, no matter how the first one turns out. Otherwise, the two things are *dependent*.

When drawing at random with replacement, the draws are independent. Without replacement, the draws are dependent.

The Chance that Both of Two Independent Things Will Happen

Special case when A and B are independent (meaning that $P(B|A) = P(B)$):

$$P(\text{both A and B}) = P(\text{A and B}) = P(A)P(B)$$

This may be stated verbally:

"If two events are independent, the chance that both will happen equals the product of their unconditional probablilites. This is a special case of the multiplication rule."

There are two important points here.

- Conditional probabilities are not needed when the events are independent.
- The chance being calculated is that both A *and* B will happen. The conjunction is '*and*'.

Mutually Exclusive: Definition

Two things are *mutually exclusive* when the occurrence of one prevents the occurrence of the other: one excludes the other.

The Addition Rule

To find the chance that at least one of two things will happen, check to see if they are mutually exclusive. If they are, add the chances.

The Chance that At Least One of Two Things Will Happen

Special case when A and B are muutally exclusive (meaning that P(A and B) = 0);

$$\text{P(at least one of A and B)} = \text{P(A or B)} = \text{P(A)} + \text{P(B)}$$

This may be stated verbally:

"If two events are mutually exclusive, the chance that either will happen equals the sum of their unconditional probablilites." This is called the "addition rule."

There are two important points here.

- When calculating the chance of B, do not assume that A has already happened: the formula does not call for the conditional probablility of B given A, but rather P(B), the unconditional chance of B happening.
- The chance being calculated is that either A *or* B will happen.

  There are several ways of stating this: 'either A or B', 'at least one of A or B', 'A or B or both', 'A and/or B', or just 'A or B'. The conjunction is always '*or*'.

The Chance that at Least One of Several Things Will Happen

Special Case: When the things are mutually exclusive.

The chance that at least one of those things will happen reduces to the chance that exactly one of them will happen—the things being mutually exclusive means that there is no chance that more than one of them will happen. This chance can be calculated by the addition rule.

The Chance of A or B, when they are Not Mutually Exclusive

If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big.

Blindly adding the chances can give the wrong answer, by double-counting the chance that both things happen. With mutually exclusive events, there is no double-counting.

There are several methods to find the chance of either of two events when the events are not mutually exclusive. One is the method of the Chevalier de Méré, presented below as the "Procedure when the things are not mutually exclusive." When applying that procedure, it is good to know that the opposite of 'A or B' is 'not A and not B,' or—as otherwise stated—the opposite of 'either A or B' is 'neither A nor B.'

The Chance that at Least One of Several Things Will Happen

(If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big. Blindly adding the chances can give the wrong answer, by double-counting chances where two or more of the things happen together. With mutually exclusive events, there is no double-counting.)

Procedure when two or more things are not mutually exclusive:

- The opposite thing is that none of those things will happen. This, the opposite thing, means that each of the original things did *not* happen. Its chance can now be calculated by repeated application of the multiplication rule. (With 2 things, only 1 application is needed.)
- The original chance that we desired—namely the chance that at least one of several things will happen—can now be found by subtracting the chance of its opposite from 100%.

Summary of de Méré procedure: A) opposite of "At least one" (same as A or B or C) is "none."
B) Find the chance of "none" (same as not A and not B and not C) using the multiplication rule.
C) Subtract that result from 1 (or 100%).

The Binomial Formula

The chance that an event will occur exactly k times out of n is given by the binomial formula

$$\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

In this formula, n is the number of trials, k is the number of times the event is to occur, and p is the probability that the event will occur on any particular trial. The assumptions:

- The value of n must be fixed in advance.
- p must be the same from trial to trial.
- The trials must be independent.

The first factor in the binomial formula is the binomial coefficient.

Chance Processes (those that produce a number as a result of chance)

In many cases the process being studied can be made analagous to the process of drawing numbers from a box. This analogy is called a box model.

The Sum of Draws

The *sum of draws* from a box is shorthand for the process of drawing tickets at random from a box and then adding up the numbers on the tickets. The applicable formula is

observed sum of the draws = expected sum + chance error.

The Expected Value for the Sum of the Draws

The expected value for the sum of draws made at random with replacement from a box equals

(number of draws) × (average of box).

The Chance Error for an Observed Sum of Draws from a Box

The chance error for an observed sum of draws made at random from a box is:

observed sum of draws − expected value for the sum of draws.

The Standard Error for the Sum of the Draws

A sum is likely to be around its expected value, but to be off by a chance error similar in size to the standard error.

When drawing at random with replacement from a box of numbered tickets, the standard error for the sum of the draws is

$$\sqrt{\text{number of draws}} \times (\text{SD of box}).$$

Using the Normal Curve to Find the Chance that the Sum of a Large Number of Draws (made at random with replacement from a box) will be in a Given Range.

First find the expected value and standard error for the sum of draws.

Next convert the endpoints of the given range to standard units using the formula

$$\text{standard units} = \frac{\text{given value} - \text{expected value}}{\text{standard error}}.$$

The area under the normal curve between the standard units for the endpoints of the given range will be an approximation for the desired chance.

A Shortcut for the SD of a Box with Only Two Kinds of Tickets

When the tickets in the box show only two different numbers, the SD of the box equals

$$\begin{pmatrix}\text{bigger} \\ \text{number}\end{pmatrix} - \begin{pmatrix}\text{smaller} \\ \text{number}\end{pmatrix} \times \sqrt{\begin{matrix}\text{fraction with} \\ \text{bigger number}\end{matrix} \times \begin{matrix}\text{fraction with} \\ \text{smaller number}\end{matrix}}$$

Classifying and Counting

If you have to classify and count the draws, put 0's and 1's on the tickets. Mark 1 on the tickets that count for you, 0 on the others.

Then find the average of the box and the expected value, and the SD (using the shortcut) and the standard error. Now convert any given values of the count to standard units and use the normal curve to approximate any chance being sought.

The Normal Approximation to Binomial Probabilities

If a binomial probability is considered as the sum of repeated draws from a suitable counting box, the normal approximation may be used—provided the number of trials (draws from the box) is suitably large.

The expected value is the product of the number of trials and the average of the counting box. The standard error is the product of the square root of the number of trials and the SD of the counting box (found by the short cut).

Since the sum of the draws is discrete, it is more accurate to correct the endpoints of the intervals by plus or minus one half.

Next convert the endpoints of the given range to standard units using the formula

$$\textbf{standard units} = \frac{\textbf{given value (corrected)} - \textbf{expected value}}{\textbf{standard error}}.$$

The area under the normal curve between the standard units for the corrected endpoints of the given range will be an approximation for the desired chance.