Solutions to Practice for Test 2, Math 125

(The test will be on April 17)

Math 125 *Kovitz* Spring 2025

On the test, show your work.

1. That means each roll was some number other than 4: chance 5/6. The ten rolls are independent, so just multiply ten factors, each 5/6. Answer: $(5/6)^{10} = 0.16$ or 16%.

   Answer: (C)

2. (A) 1/17.

   Without replacement, the draws are dependent. Use the Multiplication Rule found in the box on page 229 of the text.

   Calculating the chance using conditional probabilities:

   $$\frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4{\times}3}{17{\times}3} = \frac{1}{17}.$$

   After cancellation: 1/17.

   Answer: (A).

3. (D) 66.5%

   Do not add up six chances of getting the $\boxed{\bullet}$.

   The outcomes are not mutually exclusive; he could get the $\boxed{\bullet}$ more than once. (See the box on page 242 in the text.)

   Instead look to the opposite outcome. The opposite of getting the $\boxed{\bullet}$ at least once is not getting it at all in the six rolls. That outcome is "he got a non-$\boxed{\bullet}$ on all six rolls."

   Since the results are independent, just multiply together six factors, each of them the unconditional chance of getting a non-$\boxed{\bullet}$ (5/6). (See the box on page 232 in the text.)

   We have just found the chance of the opposite of "at least one of the six rolls results in a $\boxed{\bullet}$" to be equal to $(5/6)^6 \approx 0.335 = 33.5\%$. (Now refer to the second box on page 223 of the text.)

   The chance originally requested (of getting the $\boxed{\bullet}$ at least once) is

   $100\% - 33.5\% = 66.5\%$, answer (D).

4. (B) 2.84%

   This is a binomial probability. The clue is the word "exactly" in the question. Find n, k, and p.

   $n = 12$, $k = 5$, $n - k = 7$, $p = 1/6$, and $1 - p = 5/6$.

   The binomial formula is: The probability $= \frac{n!}{k! \times (n-k)!} p^k (1-p)^{n-k}$.

   Here $\frac{12!}{5! \times 7!} (1/6)^5 (5/6)^7 \approx 0.028425 = 2.84\%$, answer (B).

(No shortcuts or cancellation needed, just use a calculator to get the decimal answer. The subcalculation for the factorial term gives 792.)

   (The normal approximation is not that accurate here: the sample size (12) is not large.)

5. The number of draws is sufficiently large that the normal curve may be used. (Summary point 6 on page 307.)

   The method is discussed on page 294, in the first paragraph of section 17.3.

   First find the average and SD of the box. The average is the sum, 28, divided by the number of tickets, 4. The average of the box is 7.

   The deviations from the mean (x − avg) are: -3, -1, -1, and 5.

   Take their r.m.s.: $\sqrt{\frac{9+1+1+25}{4}} = \sqrt{36/4} = \sqrt{9} = 3$.

   Find the EV and SE for the sum. (summary points 2 and 3 on page 307)

   EV for the sum = $100 \times 7 = 700$.

   SE for the sum = $\sqrt{100} \times 3 = 30$.

   The standard unts for 745 = $(745 - 700)/30 = 1.50$.

   The right tail beyond 1.5 is found by subtracting the middle area and dividing by 2:

   (100%-86.64%)/2 = 6.68%, rounded to 7%.

   Answer: (C) 7%

6. (B) 8%

   The 160 draws are independent, their number is fixed at 160 in advance, there are only two possible results on each trial, and the chances are the same from trial to trial.

   The chance that the red marble will be drawn exactly 32 times out of 160 draws is given by the binomial formula. This is a binomial probability, with the clue the word "exactly."

   However, most calculators will not return the value of 160 factorial (160!).

   The normal approximation to the probability histogram for the binomial should be used, as the number of tosses is large enough.

   Set up the box model: one 1 and four 0's.

   The average of this box is (sum of tickets)/(number of tickets) = $1/5 = 0.2$.

   Its SD is found by the short-cut on page 298 to be $(1-0)\sqrt{(1/5) \times (4/5)} = 1 \times \sqrt{0.2 \times 0.8} = \sqrt{.16} = 0.4$.

   The number of heads is modeled by the sum of the draws from this 0–1 box.

   Find the EV for the sum of the draws and the SE for the sum of draws.

   They are: $160 \times 0.2 = 32$, and $\sqrt{160} \times 0.4 = 12.649(0.4) \approx 5.06$, using the box on page 289 and the lower box on page 291.

   The box of the probability histogram for 32 heads runs from 31.5 to 32.5 heads.

   (See page 317, example 1(a).) The continuity correction of the endpoints is mandatory.

   The standard units for the endpoints are:

   $(31.5 - 32)/5.06 \approx -0.10$ and $(32.5 - 32)/5.06 \approx 0.10$.

   The area under the normal curve between -0.10 and $0.10 = 7.97\% \approx 8\%$.

   There is an 8% chance tht the number of reds drawn will be exactly 32. About 8% of the people should get exactly 32 red marbles.

   Answer (B) 8%