List of Formulas, Procedures and Boxes for Test 3, May 12

(Test 3 will be on Monday, May 12, and covers material from Chapters 20, 21, 23, 24, and 26.)

Math 125 *Kovitz* Spring 2025

From Text

Box on page 298.

Box on page 301.

Boxes on pages 325 and 326.

Box on page 359 and the bottom box on page 360.

Technical note on page 362.

Boxes on pages 385 and 386.

Middle bullet on page 381.

Top half only of box on page 410.

Top box on page 412. (**Very important.**)

Technical note (ii) on the top of page 415.

Boxes on pages 416 and 422.

Procedure for confidence intervals for the population average:
last paragraph on page 416 to the top paragraph on 418.

Boxes on pages 450 and 451.

All of the discussion in 26.3, from page 478 to 481.

Boxes on pages 477, 479, 480, and 481.

Section 26.4 on page 482.

Definitions and Concepts

The Standard Error for a Percentage as the Sample Size Changes:

Multiplying the size of a sample by some factor divides the SE for a percentage not by the whole factor—but by its square root.

The Likely Result of the Draws:

When drawing at random from a box of 0's and 1's, the percentage of 1's among the draws is likely to be around its expected value, give or take its SE.

The Gauss Model

In the Gauss model, each time a measurement is made, a ticket is drawn at random with replacement from the error box. The number on the ticket is the chance error. It is added to the exact value to give the actual measurement. The average of the error box is equal to 0.

When the Gauss model applies, the SD of a series of repeated measurements can be used to estimate the SD of the error box. This estimate is good when there are enough measurements.

Formulas

Finding Areas under the Normal Curve

An area from minus a value to plus the same value is read off from the Normal Table; other areas are found by making a sketch and expressing the desired area in terms of areas that may be found by using the Table.

The Normal Approximation for Data

If a histogram follows the normal curve, approximate areas may be found by converting the endpoints to standard units and finding the appropriate areas under the normal curve by using the Table.

The Expected Value for the Sum of the Draws

The expected value for the sum of draws made at random with replacement from a box equals

$$\text{(number of draws)} \times \text{(average of box)}.$$

The Standard Error for the Sum of the Draws

A sum is likely to be around its expected value, but to be off by a chance error similar in size to the standard error.

When drawing at random with replacement from a box of numbered tickets, the standard error for the sum of the draws is

$$\sqrt{\text{number of draws}} \times \text{(SD of box)}.$$

A Shortcut for the SD of a Box with Only Two Kinds of Tickets

When the tickets in the box show only two different numbers, the SD of the box equals

$$\left(\begin{matrix}\text{bigger}\\ \text{number}\end{matrix} - \begin{matrix}\text{smaller}\\ \text{number}\end{matrix}\right) \times \sqrt{\begin{matrix}\text{fraction with}\\ \text{bigger number}\end{matrix} \times \begin{matrix}\text{fraction with}\\ \text{smaller number}\end{matrix}}$$

Classifying and Counting

If you have to classify and count the draws, put 0's and 1's on the tickets. Mark 1 on the tickets that count for you, 0 on the others.

Then find the average of the box and the expected value, and the SD (using the shortcut) and the standard error. Now convert any given values of the count to standard units and use the normal curve to approximate any chance being sought.

The Expected Value for a Sample Percentage

For a simple random sample from a given population, the following equation holds for the percentage (in a specified category)

$$\text{percentage in sample} = \text{percentage in population} + \text{chance error}.$$

With a simple random sample, the expected value for the sample percentage equals the population percentage.

The Standard Error for a Sample Percentage

$$\textbf{SE for percentage} = \left(\textbf{SD of box}/\sqrt{\textbf{number of draws}}\right) \times \textbf{100}\%.$$

It is a fact that the SD of a 0–1 counting box is always .5 or less. This means that the SE for a percentage is always less than $\dfrac{50\%}{\sqrt{\text{number of draws}}}$, no matter what the percentage of the box.

Warning: if the problem involves classifying and counting to get a percent, put 0's and 1's in the box.

Confidence intervals

A confidence interval for an average—with a confidence level specified as a percent—is a range such that you are that percent confident that the population average is in that interval.

A confidence interval is based on the results of a single sample of a particular size.

- the interval "sample average ± 2 SEs" is an approximate 95%-confidence interval for the population average.
- the interval "sample average ± 3 SEs" is an approximate 99.7%-confidence interval for the population average.

The EV and SE for the Average of the Draws

When drawing at random from a box:

$$\text{EV for average of draws} = \text{average of box.}$$

$$\text{SE for average of draws} = \frac{\text{SE for sum}}{\text{number of draws}}.$$

If you wish, you may compute the SE for an average directly from the SD of the box by the formula

$$\text{SE for average} = (\text{SD of box})/\sqrt{\text{number of draws}}.$$

Using the Normal Curve to get Chances for the Average of a Large Number of Draws

When drawing at random from a box, the probability histogram for the averge of the draws follows the normal curve, even if the contents of the box do not. The histogram must be put into standard units, and the number of draws must be reasonably large.

The formula for the standard units for a given average is:

$$\frac{\text{observed average} - \text{EV for the sample averge}}{\text{standard error for the sample average}}.$$

Do not make the common mistake of using the SD instead of the SE in the denominator. True, the average of the original box and the expected value for sample average are equal, but their deviations are not.

Various Standard Errors

For a given box model there are several SEs, each showing the likely size of a certain chance error. The corresponding formulas are:

$$\begin{aligned}
\text{SE for sum} &= \sqrt{\text{number of draws}} \times \text{SD of box} \\
\text{SE for average} &= \frac{\text{SE for sum}}{\text{number of draws}} = \frac{\text{SD of box}}{\sqrt{\text{number of draws}}} \\
\text{SE for count} &= \text{SE for sum, from a 0–1 box} \\
\text{SE for percent} &= \frac{\text{SE for count}}{\text{number of draws}} \times 100\% = \frac{\text{SD of zero-one box}}{\sqrt{\text{number of draws}}} \times 100\%
\end{aligned}$$

The SE for the sum is basic; the other formulas all come from that one. These formulas apply to draws made at random with replacement from a box.

Do not confuse the SD and the SE for the average.

- The SD says how far a number in the box is from average—for a typical number.
- The SE for the average says how far the sample average is from the population average—for a typical sample.

The Gauss Model for Measurement Error

The Gauss model applies to repeated measurements on some quantity. According to the model, each time a measurement is made, a ticket is drawn at random with replacement from the error box. The number on the ticket is the chance error. It is added to the exact value to give the actual measurement. The average of the error box is equal to 0.

Since the average of the error box is 0 and each measurement is the exact value (a constant) added to the ticket from the error box, the population average weighing equals the exact value.

When two quantities differ by a constant value, they have the same SD. So the SD of the error box is the same as the SD of the measurements. When the Gauss model applies, the SD of a series of repeated measurements can be used to estimate the SD of the error box. The estimate is good when there are enough measurements.

Now, the sample average of the measurements will be off from the population average (the exact value) by the standard error for the sample average. This standard error is found by dividing the estimated SD of the error box by the square root of the size of the sample.

Tests of Significance: The Null and the Alternative

To make a test of significance, the null hypothesis has to be formulated as a statement about a box model. Usually, the alternative does too.

- The *null hypothesis* says that an observed difference just reflects chance variation.
- The *alternative hypothesis* says that the observed difference is real.

The null hypothesis expresses the idea that an observed difference is due to chance. To make a test of signficance, the null hypothesis has to be set up as a box model for the data. The alternative hypothesis is another statement about the box; it says that the difference is real.

Testing Hypotheses: The Observed Significance Level

A test statistic is used to measure the difference between the data and what is expected on the null hypothesis.

z says how many SEs away an observed value is from its expected value, where the expected value is calculated using the null hypothesis. The formula is

$$z = \frac{\text{observed value} - \text{expected value}}{\text{standard error}}.$$

> The observed significance level—often called the P-value—is the chance of getting a test statistic as extreme as or more extreme than the observed one. The chance is computed on the basis that the null hypothesis is right. The smaller the chance is, the stronger the evidence against the null.
>
>> A small value of P indicates that an explanation saying that this is chance variation is unreasonable. We cannot accept the model stated in the null.
>>
>> A large value of P could very well be due to chance variation and we accept the null as a reasonable model.
>
> The P-value of a test is the chance of getting a big test statistic—assuming the null hypothesis to be right. P is not the chance of the null hypothesis being right.

Making a Test of Significance

Based on some available data, the investigator has to—

- translate the null hypothesis into a box model for the data;
- define a test statisic to measure the difference between the data and what is expected on the null hypothesis;
- compute the observed significance level P.

The choice of test statistic depends on the model and the hypothesis being considered.

Making the Decision

Many statisticians have a dividing line that indicates how small the observed significance level has to be before an investigator should reject the null hypothesis.

- If P is less than 5%, the result is called *statistically significant* and the null hypothesis is rejected.

- If P is greater than 5%, we accept the null and state that chance error is a reasoanable explanation for this result.

Tests of Significance with Zero-one Boxes

When the situarion involves classifying and counting, the z-test can also be used. Just put 0's and 1's in the box.

If n trials were run, the null hypothesis says that the data can be considered as a sum of n draws from the given 0–1 box. That counts up the number of times that the result of interest has occurred. The normal approximation applies if the number of draws is large, and the z-test can be used.

Be careful, when finding z (the standard units), to use the **SE for the SUM.**

With a one-tailed test, P will be the area of the pertinent tail.

In counting, the continuity correction might be used to take care of the endpoints (see page 317.). However, with a large sample and normal accuracy desired, that $\pm 1/2$ adjustment is often ignored.