Solutions to Sample of Final Exam Problems

(Final on May 23)

Math 125, Spring 2025

Multiple Choice. $3\frac{1}{2}$ points for each correct response, no points deducted for a wrong answer

1. The percentage for the block with base \$75,000 to \$90,000 is the producet of the width of the base and the height of the block.

   Here: (\$90,000-\$75,000) $\times$ 2% per \$1,000 = \$15,000 $\times$ 2%/\$1,000 = 30%. Answer: (C)

   For a rectangle: AREA = (LENGTH times WIDTH) or (BASE times HEIGHT).

2. The average $x = \frac{4+11+13+13+14+17}{6} = 72/6 = 12$.

   The SD of $x =$

$$\sqrt{\frac{(4-12)^2+(11-12)^2+(13-12)^2+(13-12)^2+(14-12)^2+(17-12)^2}{6}}$$

$$= \sqrt{\frac{64+1+1+1+4+25}{6}} = \sqrt{\frac{96}{6}} = \sqrt{16} = 4$$

   .

   Note: The alternative way on page 74 gives

$$\sqrt{\frac{16+121+169+169+196+289}{6} - 13^2} = \sqrt{\frac{960}{6} - 144} = \sqrt{160-144} = \sqrt{16} = 4.$$

   Answer: (B) 4

3. Answer: (D)

   The range 300–700 correpsonds to ± 2 in standard units. About 95.45% of the students at the university had scores in this range.

   Let n be the number of students at the university. One thousand divided by the total number of students is 0.9545. (That fraction will end up 95.45% when multiplied by 100%.)

   So $1000/n \approx 0.9545$. Solving for n, $n = 1000/0.9545 = 1048$. There must have been about 1048 students at the university.

   The range 400–600 corresponds to ± 1 in standard units (It's half as wide.). About 68.27% of the students had score in htis range: 68.27% of 1048 = $0.6827 \times 1048 \approx 715$. Anser: (D)

   Moral: the normal curve is not a rectangle.

   Note: this problem could also be done more directly by noting that .6827/.9545 of the 1000 in the larger range would end up in the smaller range.

   Then $1000 \times (0.6827/0.9545) \approx 715$.

4. First convert the score to standard units: $(315-500)/100 = -185/100 = -1.85$

   Then note that the area below -1.85 on the normal curve will be the precentile rank.

   First look up 1.85 to see that the area in the middle is 94%.

   We want the area of the left-hand tail. Take 94% from 100% to get 6%, then divide the 6% by 2:

   Answer: (C) the 3rd

5. (C) 0.375

   First find the average x and the average y:

   avg $x = \frac{6+8+2+10+14}{5} = \frac{40}{5} = 8$ and avg $y = \frac{3+11+12+9+15}{5} = \frac{50}{5} = 10$.

   Then find the SD of x and the SD of y:

   SD of $x = \sqrt{\frac{(-2)^2+0^2+(-6)^2+2^2+6^2}{5}} = \sqrt{\frac{80}{5}} = 4$. Same for SD of y; it equals 4.

   Now convert each of the ten numbers in the original table to standard units, and put the ten results in a new table.

   Proceed like this: $\frac{6-8}{4} = \frac{-2}{4} =$ -0.5.

   The ten results are put in the new table and each row is multiplied across to get the products.

x std. units	y std. units	product
-0.5	-1.75	0.875
0	0.25	0
-1.5	0.5	-0.75
0.5	-0.25	-0.125
1.5	1.25	1.875

   Then get the average of the 5 products. They add to 1.875; divide by 5: 0.375.

   That is r. The correlation coefficient, r, for the above data set is: (C) 0.375

   Alt. Method to get SD for x (top of pg. 74): avg of squares is (36+64+4+100+196)/5=400/5=80.

   Then the square of the average is $8^2 = 64$. SD $= \sqrt{80-64} = \sqrt{16} = 4$.

   Alt. Method to get SD for y (top of pg. 74): avg of squares is (9+121+144+81+225)/5=580/5=116.

   Then the square of the average is $10^2 = 100$. SD $= \sqrt{116-100} = \sqrt{16} = 4$.

   Alt. Method to get r (pg. 134, Techincal Note): avg of products is (18+88+24+90+210)/5=430/5=86.

   Then the product of the averages is $8 \times 10 = 80$. $r = (86-80)/(\text{SD of } x \times \text{SD of } y) = 6/(16) = 0.375$.

6. Answer: (B)

   Reasons:

   (A) False. The correlation is negative so heavier persons tend ot be less educated.

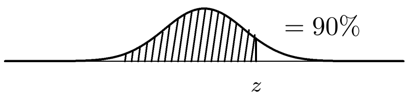
   (B) True. With negative correlation, as one variable increases, the other decreases.

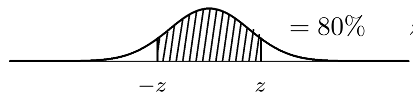
   (C) False. Switching the given and the predicted will not change the ocrrelation coefficient. It will stay -0.10.

   (D) False. Change of scale will not alter the standard units. The correlation ocefficeint remains the same.

   (E) False. Correlation is not causation.

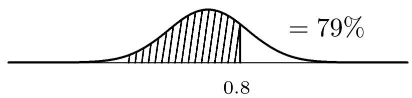
7. Answer: (C) 79%

Work: 



$z \approx 1.30$

In standard units, his SAT score was 1.30. The regression prediction for his first-year score is $0.6 \times 1.30 = 0.78 \approx 0.80$ in standard units.



This corresponds to a predicted percentile rank of 79% for that student's GPA.

Details of converting percentile to z: Take the given 90% as the area below z. That means that the upper tail must be 10% and the so-far-unpictured lower tail must be 10%. Take the lower tail from the given percentile rank and you will have the percentage value that must be looked up on the table: 80%. That gives $z = +1.30$, because we want the value that is positive.

It is also possible to do the first step with a slightly different logic. Take the 90%, the given percentile rank and break it into 2 parts: the area below 0 (50%), and the area between 0 and z (40%). Then double the 40% to get the middle area (from $-z$ to z) for looking up on the table. Again look up 80% on the normal table and find z.

Details of converting z to a percentile, as required in the last step: $z = 0.8$ has 58% in the middle. Below it will be the 50% below 0 and the 29% from 0 to z. (This 29% is half of the middle area for $z = 0.80$.) The sum is 79%. That's the area below z, so by definition it is the percentile rank.

Alternative logic: 58% in the middle, means 100% − 58% = 42% for the two tails. One will be 21%. We need all area except one tail = 100% − 21% = 79%.

This corresponds to a predicted percentile rank of 79% for that student's GPA. (Usually round off to nearest whole percentage.)

8. This GPA is above average, by some number of SDs. We need to find that number, call it z. Draw the normal curve and shade in the appropriate area. It is clear that z is to the right of 0, hence a positive value.

   The normal table cannot be used directly, because it gives the area between $-z$ and z, rather than the area to the left of z.

   The 84th percentile means that 84% of the area is below z. Break that area up into two parts: the portion below 0 and the portion between 0 and z. Below 0 is 50%, so that leaves 34% between 0 and z. The area between $-z$ and 0 is also 34%, by symmetry. Look up the area from $-z$ to z, 68%, on the normal table. The value of z is 1.

   With $z = 1$, the standard units for the 84th percentile GPA are +1.

   Using the box on page 79, the GPA is 1 SD above average: average + 1SD = 2.6 +(1)0.6 = 3.2.

   Answer: (D) 3.2

9. See Figure 3 on page 183. About 68% of the points will fall inside the strip whose edges are parallel to the regression line and one r.m.s. error away (up or down).

   The r.m.s. error is

   $\sqrt{1-r^2} \times (\text{the SD of y}) = \sqrt{1-.6^2} \times 15 = \sqrt{1-.36} \times 15 = \sqrt{0.64} \times 15 = 0.8 \times 15 = 12.$

   Answer: (C)

10. That means each roll was some number other than 4: chance 5/6. The twenty rolls are independent, so just multiply twenty factors, each 5/6. Answer: $(5/6)^{20} = 0.026$ or 2.6%.

    Answer: (B)

11. (D) 65%

    Do not add up four chances of hitting the bullseye. The outcomes are not mutually exclusive; he could hit the bullseye more than once. (See the box on page 242 in the text.)

    Instead look to the opposite outcome. The opposite of hitting the bullseye at least once is not hitting it at all in the four throws. That outcome is "he missed the bullseye on all four throws."

    Since the results are independent, just multiply together four factors, each of them the unconditional chance of missing (0.77). (See the box on page 232 in the text.)

    We have just found the chance of the opposite of "at least one of the four throws results in a bullseys" to be equal to $(0.77)^4 \approx 0.35 = 35\%$. (Now refer to the second box on page 223 of the text.)

    The chance originally requested (of hitting the bullseye at least once) is

    $100\% - 35\% = 65\%$, answer (D).

12. (F) 42%

    This is a binomial probability. The clue is the word "exactly" in the question. Find n, k, and p.

    $n = 4$, $k = 1$, $n - k = 3$, $p = 0.23$, and $1 - p = 0.77$.

    This is hitting the bullseye once, so $k = 1$ and $p = 0.23$, the chance of hitting the bullseye on each throw.

    The binomial formula is: The probability $= \frac{n!}{k! \times (n-k)!} p^k (1-p)^{n-k}$. (page 259)

    Here $\frac{4!}{1! \times 3!} (0.23)^1 (0.77)^3 \approx 0.42 = 42\%$, answer (F).

13. (B) 2/17.

    Without replacement, the draws are dependent. Use two applications of the Multiplication Rule found in the box on page 229 of the text.

    Calculating the chance using conditional probabilities:

    $$\frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{1}{2} \times \frac{25}{17 \times 3} \times \frac{3 \times 2 \times 2 \times 2}{25 \times 2}.$$

    After cancellation: 2/17.

    Answer: (B).

14. Since the tickets are drawn without replacement, conditional probabilities will be necessary. Use two applications of the multiplication rule, found in the box on page 229.

    If the two tickets left are the 4 and the 5, the three tickets drawn must be the 1, 2, and 3.

    Find the chance that the first draw is 1, 2, or 3; the second draw is 1, 2, or 3; and the third draw is 1, 2, or 3—with conditonal chances for the second and third draws.

    The probability will be: $(3/5) \times (2/4) \times (1/3) = 1/10$ after cancellation.

    Answer: (A) 1/10

15. This is a binomial with $n = 2$, $k = 1$, $n - k = 1$, $p = 1/4$, and $1 - p = 3/4$.

    The chance is: $\frac{2!}{1! \times 1!} \times (1/4)^1 \times (3/4)^1 = 2 \times (1/4) \times (3/4) = 6/16 = 3/8$.

    Answer: (C) 3/8

16. (B) 3.01%

    This is a binomial probability, with the clue the word "exactly."

    However, most calculators will not return the value of 350 factorial (350!).

    The normal approximation to the probability histogram for the binomial should be used, as the number of tosses is large enough.

    Set up the box model: one 0 and one 1. The average and the SD of this box are both 1/2.

    The number of heads is modeled by the sum of the draws from this 0–1 box.

    Find the EV for the sum of the draws and the SE for the sum of draws.

    They are: $350 \times 0.5 = 175$, and $\sqrt{350} \times 0.5 = 9.354$.

    The box of the probability histogram for 182 heads runs from 181.5 to 182.5 heads.

    (See page 317, example 1(a).)

    The standard units for the endpoints are:

    $(181.5 - 175)/9.354 \approx 0.70$ and $(182.5 - 175)/9.354 \approx 0.80$.

    The area under the normal curve between 0.70 and 0.80 = 57.63%/2 − 51.61%/2 = 3.01%.

17. The number of draws is sufficiently large that the normal curve may be used. (Summary point 6 on page 307.)

    The method is discussed on page 294, in the first paragraph of section 17.3.

    First find the average and SD of the box. The average is the sum, 42, divided by the number of tickets, 7. The average of the box is 6. Or, just note that the box is symmetric, so the average equals the median: 6.

    The deviations from the mean (x − avg) are: −3, −2, −1, 0, 1, 2, and 3.

    Take their r.m.s.: $\sqrt{\frac{9+4+1+0+1+4+9}{7}} = \sqrt{28/7} = \sqrt{4} = 2$.

    Find the EV and SE for the sum. (summary points 2 and 3 on page 307)

    EV for the sum = $50 \times 6 = 300$.

    SE for the sum = $\sqrt{50} \times 2 \approx 14.142$.

    The standard unts for 327 = $(327 - 300)/14.142 = 1.90$.

    The right tail beyond 1.9 is found by subtracting the middle area and then dividing by 2: (100%-94.26%)/2 = 2.87%.

    Answer: (A) 2.87%

18. (D) 3.44%

    To get the proper counting box needed to model the percentage, put 1 on the even-numbered cards and put 0 on all the other cards.

    The give or take for the sample percentage is just the SE for the sample percentage, which has the formula: $\dfrac{\text{SD of the box}}{\sqrt{\text{no. of draws}}} \times 100\%$.

    (See the technical note on page 362 of the text.)

    Use the shortcut to find the SD of the box:

    The fraction of tickets with 1 is 20/52, and the fraction of tickets with 0 is 32/52.

    The formula gives: $(1 - 0)\sqrt{\frac{32}{52} \times \frac{20}{52}} = \sqrt{0.38462 \times 0.61538} = \sqrt{0.23669} = 0.4865$.

    (See the formula on page 298.)

    Then the answer is: $\frac{0.4865}{\sqrt{200}} \times 100\% = 3.44\%$, answer (D).

19. (B) 64.4 to 67.6%

    (a) Set up a 0–1 box where 1 is on the tickets for readers of newspapers and 0 on the others.

    (b) Find the sample percent: $(2310/3500) \times 100\% = 66\%$.

    (c) Use the bootstrap to estimate the fraction of 1's in the box: 2310/3500 or 0.66. The fraction of 0's in the box will be 0.34.

    (d) Estimate the SD of the box by the shortcut: $\sqrt{0.66 \times 0.34} = \sqrt{0.2244} = 0.4737$.

    (e) Estimate the SE for the percentage: $\frac{\text{SD of box}}{\sqrt{\text{no. of draws}}} \times 100\% = \frac{0.4737}{\sqrt{3500}} \times 100\% = \frac{47.37\%}{59.161} = 0.8\%$.

    (f) Write 95%-confidence interval as sample percent plus or minus two SEs.

    (g) Answer is: $66\% \pm 1.6\%$. That's 66%-1.6% to 66%+1.6%, or 64.4% to 67.6%, answer (B).

20. (D) 82.30%

    The average of the draws is the same as the sample average.

    The expected value for the average of the draws is equal to the average of the box, here 120. (See the box on page 410.)

    The standard error for the average is (using the technical note on page 415) the SD of the box, divided by the square root of the number of draws.

    Here it gives: $20/\sqrt{81} = 20/9 \approx 2.2222$.

    A sample of size 81 should be large enough for the normal approximation to apply.

    The standard units for a given sample average is found with the formula:

    $$z = \frac{\text{given sample average} - \text{EV for sample average}}{\text{SE for sample average}}.$$

    Find the standard units for 117 and 123: $(117 - 120)/2.2222 = -1.35$ and $(123 - 120)/2.2222 = 1.35$.

    The area under the normal curve from -1.35 to $+1.35$ is just the table entry: 82.30%, answer (D).

21. Answer: (C) 15

    See the second box on page 412, the square-root law.

22. Answer: (D) Each measurement will be off the the SD or so, not by the SE.

23. Rolling the die is like drawing at random with replacement 1620 times from a 0–1 box, with 1 on threes and 0 on the rest. The fraction of 1's in the box is unknown.

    Null hypothesis: this fraction equals 1/6.

    Alternative: the fraction is bigger than 1/6. (We observe that 283 is somewhat larger than 270, the expected value.)

    The number of three spots is like the sum of the draws.

    $z = 0.83$, $P \approx 20\%$, fair.

    Calculation: The average of the box = 1/6 and the SD of box = $\sqrt{1/6 \times 5/6} = \sqrt{5}/6 \approx 0.372678$.

    So the EV sum $= 1620 \times 1/6 = 270$ and the SE sum $= \sqrt{1{,}620} \times \sqrt{5}/6 = \sqrt{8100}/6 = 90/6 = 15$.

    Using decimals, the SE for the sum $= \sqrt{1620} \times 0.372678 \approx 15$. The decimals introduce a tiny round-off error.

    Then $z = \dfrac{\text{observed sum - EV sum}}{\text{SE sum}} = \dfrac{282.5 - 270}{15} = 0.83.$

    Ignoring the continuity correction would give $(283 - 270)/15 = .87$, and hardly any difference in P.

    To get P, the area of the right tail, look up 0.85 and subtract that 60.47% from 100% to get 39.53%. Then divide by 2 to get 19.76%.

    This looks like chance variation. That large significance level P (more than 5%) makes an explanation of the null with chance error reasonable.

    It looks like the die is fair. Answer: (C)

24. First state the null and alternative hypotheses.

    Null: The average of the box equals 20.

    Alternative: The average of the box is more than 20.

    Now, assume that the null is true, and decide if a result of the observed average or more extreme should be attributed to chance.

    Because the draws are made at random from a box, the square root law applies. (Second box on page 291, applied to averages.) And because the number of draws is reasonably large, the normal curve may be used to get chances for the average of the draws. (See the box on page 412.)

    Since the null hypothesis is assumed to be true, the EV for the sample average (average of the draws) is 20.

    Now find the SE for the average of the draws.

    First estimate the SD of the box by the SD of the sample, which is 10. (Box on page 416)

    Then use the formula for the SE for the average of the draws found on the top of page 415: $10/\sqrt{100} = 20/10 = 1$.

    The standard units come out to be:

    $$\frac{\text{observed average} - \text{EV for average of draws}}{\text{SE for average of draws}} = \frac{22.7 - 20}{1} = 2.27.$$

    P will be the area of the right tail: $(100\% - 97.56\%)/2 = 0.44\%/2 = 1.22\%$.

    Since P is less than 5%, conclude that chance error is not a reasonable explanation and the average of the box is more than 20. Answer: (B)

25. (A) 1%, unfair

There are 5 degees of freedom: one less than the number of categories.

The expected values are all $600 \times (1/6)$, or 100.

For each category, subtract the expected value from the observed, square the result, and then divide by 100, the expected.

Here is the first calculation: $\frac{(87-100)^2}{100} = 169/100 = 1.69$.

Add the five results. This is χ^2: $1.69 + 2.25 + 0.36 + 6.25 + 4.84 + 0.01 = 15.4$.

Or, since they are all over 100, add the six squares of the (observed − expected) and then divide by 100: $\frac{169+225+36+625+484+1}{100} = \frac{1540}{100} = 15.4$.

From the χ^2-table, the value of P is estimated at 1%. It's a bit less than 1%.

Since P is less than 1%, we do not attribute the difference to chance error. Instead, we reject the null hypothesis and conclude that the die is unfair.

Answer: (A) 1%, unfair