Supplementary Practice for the Final Examination

(Final on Dec. 15)

Math 125, Fall 2023

Multiple Choice. 5 points for each correct response, 1 point deducted for each wrong answer

1. A distribution of families by size had a total number in the survey of 12,000 families and 414 families of size 6 to 8, inclusive.

   A histogram was drawn for that distribution. For a block representing the familiies of size 6 to 8 inclusive, state its leftmost point and its height.

   (For height, just give the value, ignoring units. The units are percent per width of the base.)

   (A) 5.5, 1.15 (B) 5.5, 1.725 (C) 5.5, 3.45 (D) 6, 3.45 (E) 6.5, 1.725

2. A list has 3 entries: -1, -1, and 2. The SD of that list is:

   (A) 1 (B) 4/3 (C) $\sqrt{2}$ (D) $\sqrt{5/3}$ (E) 3/2

3. A list has 3 entries: -1, -1, and 2. In standard units, the value 2 is:

   (A) 1 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\frac{2}{5}\sqrt{15}$ (E) 2

4. A list has 5 entries: $-1/2$, $-1/2$, $-1/2$, $-1/2$, and 2. The SD of that list is:

   (A) 1/2 (B) 1 (C) 4/3 (D) $\sqrt{2}$ (E) 3/2

5. The heights of a large group of women follow the normal curve. One woman is 1.35 SDs above average.

   What percent of the group had heights equal to her or less?

   (A) 9% (B) 18% (C) 41.15% (D) 50% (E) 82.30% (F) 91%

For questions 6 and 7:

The heights of a large group of women follow the normal curve with an average of 65 inches and an SD of 2.5 inches.

6. The percent of women who are between the 95th percentile height and the 97th percentile height is ____________ the percent of women who are between the 75th percentile height and the 77th percentile height.

   (A) greater than (B) equal to (C) less than (D) indeterminable in relation to (E) (don't know)

7. The difference between the 60th percentile height and the 5th percentile height is:

   (A) 0.625 inches (B) 2 inches (C) 2.875 inches (D) 3.5 inches (E) 4.75 inches

8.

x	y
1	3
2	2
3	1

The correlation coefficient, r, for the above data set is:

(A) -1 (B) $-3/4$ (C) $-1/2$ (D) 0 (E) 1

For problems 9, 10, 11, and 12

A statistical analysis was made of the midterm and final exam scores in a large course, with the following results:

average midterm score = 50 points, SD = 25 points
average final exam score = 55 points, SD = 15 points, $r = 0.60$

9. Predict the final exam score for a student whose midterm score was 80 points.

(A) 60.8 points (B) 65.8 points (C) 70 points (D) 73 points (E) 85 points

10. This prediction is likly to be off by ________ points or so.

(A) 6 (B) 9 (C) 12 (D) 15 (E) 25

11. Find the regression equation for predicting final exam score from midterm score.

(A) final exam score = midterm score + 5
(B) final exam score = midterm score − 5
(C) final exam score = 0.36 × midterm score + 35.2
(D) final exam score = 0.36 × midterm score + 37
(E) final exam score = 0.6 × midterm score + 25

12. Use the equation to predict the final exam score for a student whose midterm score was 80 points. Show the calculation resulting from plugging in the given value.

(A) $0.36(80) + 35.2 = 64$ (B) $0.36(80) + 37 = 65.8$ (C) $0.6(80) + 25 = 73$
(D) $80 - 5 = 75$ (E) $80 + 5 = 85$

For problems 13, 14, 15, 16, 17, 18, 19, 20, 21, and 22

The relationship between height and weight for a group of women is summarized as follows:

average height = 65 inches, SD = 3 inches
average weight = 130 pounds, SD = 12 pounds, $r = 0.40$

13. Predict the weight of a 71-inch-tall woman.

    (A) 132.4 pounds (B) 134.8 pounds (C) 136 pounds (D) 139.6 pounds (E) 154 pounds

14. This prediction is likly to be off by ______ pounds or so.

    (A) 1.2 (B) 2.75 (C) 4.8 (D) 10.08 (E) 11

15. Find the regression equation for predicting weight from height.

    (A) weight = 0.1 × height + 123.5 (B) weight = 0.25 × height + 113.75
    (C) weight = 0.8 × height + 78 (D) weight = 1.6 × height + 26
    (E) weight = 4 × height − 130

16. Use the equation to predict the weight for a woman whose height was 71 inches. Show the calculation resulting from plugging in the given value.

    (A) $0.1(71)+123.5 = 130.6$ (B) $0.25(71)+113.75 = 131.5$ (C) $0.8(71)+78 = 134.8$
    (D) $1.6(71) + 26 = 139.6$ (E) $4(71) - 130 = 154$

17. Predict the height of a 150-pound woman.

    (A) 65.667 inches (B) 67 inches (C) 70 inches (D) 73 inches (E) 74.6 inches

18. This prediction is likly to be off by ______ inches or so.

    (A) 1.2 (B) 2.75 (C) 4.8 (D) 10.08 (E) 11

19. Find the regression equation for predicting height from weight.

    (A) height = 0.1 × weight + 52 (B) height = 0.25 × weight + 32.5
    (C) height = 5/8 × weight − 16.25 (D) height = 0.8 × weight − 39
    (E) height = 1.6 × weight − 143 (F) height = 4 × weight − 455

20. Use the equation to predict the height for a woman whose weight was 150 pounds. Show the calculation resulting from plugging in the given value.

    (A) $0.1(150) + 52 = 67$ (B) $0.25(150) + 32.5 = 70$ (C) $(5/8)150 - 16.25 = 77.5$
    (D) $0.8(150) - 39 = 81$ (E) $1.6(150) - 143 = 97$ (F) $4(150) - 455 = 145$

21. Women who were 73 inches tall were estimated to average 142.8 pounds, but much heavier women who weighed 160 pounds were estimated to average only 68 inches in height.

    Shouldn't they have been more than 73 inches tall on average? After all they were heavier than the 142.8 pounds, the estimated average weight of the 73 inch tall women. Explain.

    (A) The method of finding the estimated averages was wrong.

    (B) It's just the regression effect (regression is toward the mean) and the regression fallacy.

    (C) All is fine; there are two different regression lines, depending on which variable is given.

22. A woman who weighed 140 pounds and was 67 inches tall gained 20 pounds to now weigh 160 pounds. What's her new height?

    (A) 65 inches (B) 66.2 inches (C) 67 inches (D) 70 inches (E) 71 inches

23. Suppose for another set of data the regression equation for predicting weight from height was: weight $= 2 \times$ height $+ 10$.

    True or false and explain: The regression equation for predicting height from weight will be: height $= 1/2 \times$ weight $- 5$, because all you have to do is solve algebraically for the other variable in the equation for weight from height.

    (A) True.

    (B) False. There are two different equations, depending on which variable is given.

24. For a group of men, weights are predicted from heights using the regression method. The correclation coefficient is 0.62 and the SD of the weights is 27.78 pounds.

    About what percent of the time will the predictions be right within 10.9 pounds?

    (A) 31% (B) 38% (C) 68% (D) 95% (E) 99%

25. The heights and weights of a large group of women followed the normal curve. The averages and SDs are not known. However, the correlation coefficient r is around 0.49.

    A woman with height at the 91st percentile should have weight around what percentile?

    (A) 50th (B) 60th (C) 70th (D) 74th (E) 82nd (F) 91st

26. A fair die is rolled four times. To determine the chance of rolling at least one five spot, note that the sum of the four individual chances is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

    The chance of rolling at least one five must be ____________ 2/3.

    (A) less than (B) equal to (C) greater than (D) uncomparable to (E) don't know

27. A box has six tickets, numbered 1 to 6. Four tickets will be drawn without replacement. To determine the chance of drawing at least one ticket numbered 5, note that the sum of the four individual chances is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

    The chance of drawing at least one five must be ____________ 2/3.

    (A) less than (B) equal to (C) greater than (D) uncomparable to (E) don't know

For questions 28 and 29

A box contains five tickets: one red and four blue. Two tickets are to be drawn at random from the box. Find the chance of drawing at least one red ticket.

In each question (28 and 29) select a letter from the 6 (A to F) that has the correct chance and a valid explanation. Multiple answers may be right; select one of the correct letters.

28. The draws are made without replacement.

29. The draws are made with replacement.

(A) 45%. Drawing at least one red ticket means getting a red ticket on the first draw or the second draw. The results are mutually exclusive; add the chances. The chance of a red is $1/5 = 20\%$ on the first draw and $1/4 = 25\%$ on the second draw. The total is 45%.

(B) 40%. Drawing at least one red ticket means getting a red ticket on the 1st draw or the 2nd draw. The results are mutually exclusive; add the unconditional chances. The chance of a red is $1/5 = 20\%$ on the 1st draw and $1/5 = 20\%$ on the 2nd draw. The total is 40%. (Note: the chance of red on the second draw is 20%, not 25%. Study carefully example 2(a) on page 226.)

(C) 36%. Getting red on each draw is not mutually exclusive, so do not add the chances. Instead find the chance of the opposite event and subtract it from 100%. The opposite of at least one red is no reds: chance $(4/5) \times (4/5) = 16/25 = 64\%$. Answer:$100\% - 64\% = 36\%$.

(D) 40%. Find the chance of the opposite event, which is getting no reds. Without replacment, conditional probabilities are needed: $(4/5) \times (3/4) = 3/5 = 60\%$. The original chance is found by subtracting from 100%: $100\% - 60\% = 40\%$.

(E) 36%. Getting at least one red is getting (1) a red on the first or (2) a blue on the first and a red on the second. Outcomes (1) and (2) are mutually exclusive and cover the result that was specified. So add P(red) = 20% and P(blue, then red) $= (4/5) \times (1/5) = 4/25 = 16\%$. The answer is 36%. (With replacement and independence, the chances for the second draw are 1/5.)

(F) 40%. Getting at least one red is getting (1) a red on the first or (2) a blue on the first and a red on the second. Outcomes (1) and (2) are mutually exclusive and cover the result that was specified. So add P(red) = 20% and P(blue, then red) $= (4/5) \times (1/4) = 1/5 = 20\%$. The answer is 40%. (There is no replacement, so the chances for the second draw are 1/4.)

30. Assume that the chances of Boston's teams winning the league championship in the next cycle are: Red Sox 1/6, Patriots 1/3, and Bruins 1/4, and that the chances are independent.

With those assumptions, the chance that Boston would—in the next cycle—have at least one championship team among the three is found by a valid formula to be:

(A) $\frac{1}{6} + \frac{1}{3} + \frac{1}{4} = \frac{3}{4}$ (B) $1 - (\frac{5}{6} \times \frac{2}{3} \times \frac{3}{4}) = 1 - \frac{5}{12} = \frac{7}{12}$ (C) $\frac{5}{6} \times \frac{2}{3} \times \frac{3}{4} = \frac{5}{12}$

(D) $1 - (\frac{1}{6} + \frac{1}{3} + \frac{1}{4}) = 1 - \frac{3}{4} = \frac{1}{4}$ (E) $1 - (\frac{1}{6} \times \frac{1}{3} \times \frac{1}{4}) = 1 - \frac{1}{48} = \frac{47}{48}$

For questions 31 to 33:

A quiz contains six multiple-choice questions. Each question has five possible answers, only one of which is correct.

A student answers the six questions randomly.

(In each of problems 31, 32 and 33, estimate the chance to the nearest 0.1%.)

31. The chance that the student gets at least one question correct is about:

    (A) 0% (B) 1.5% (C) 26.2% (D) 39.3% (E) 73.8%

32. The chance that the student gets exactly one question correct is about:

    (A) 0% (B) 1.5% (C) 26.2% (D) 39.3% (E) 73.8%

33. The chance that the student gets at least two questions correct is about:

    (A) 0% (B) 1.5% (C) 26.2% (D) 34.5% (E) 39.3%

For questions 34 to 36:

A standard deck of cards has 52 cards: 36 numbered cards and 16 face cards.

A player draws four cards without replacement.

34. The chance that the player gets all numbered cards is about:

    (A) 0.67% (B) 0.90% (C) 21.76% (D) 22.97% (E) 78.24% (F) 99.10% (G) 99.33%

35. The chance that the player gets at least one face card is about:

    (A) 0.67% (B) 0.90% (C) 21.76% (D) 22.97% (E) 78.24% (F) 99.10% (G) 99.33%

36. The chance that the player gets at least one numbered card is about:

    (A) 0.67% (B) 0.90% (C) 21.76% (D) 22.97% (E) 78.24% (F) 99.10% (G) 99.33%

37. A standard deck of cards has 52 cards: 36 numbered cards and 16 face cards.

    A player draws four cards with replacement.

    The chance that exactly two of the four cards drawn are numbered is about:

    (A) 4.538% (B) 6.65% (C) 18.15% (D) 27.23% (E) 44.04% (F) 46.6%

For questions 38 to 40

A standard deck of cards has 52 cards: 36 numbered cards and 16 face cards.

A player draws one hundred cards with replacement.

38. The expected percentage of numbered cards drawn is about:

    (A) 0.692% (B) 21.30% (C) 30.77% (D) 44.44% (E) 69.23%

39. The give or take for the expected percentage of numbered cards drawn is about:

    (A) 2.130% (b) 3.077% (C) 4.615% (D) 6.923% (E) 21.30%

40. The probability that the perecent of numbered cards in the one hundred cards drawn is between 60% and 80% is approximately:

    (A) 36.51% (B) 85.1% (C) 96.8% (D) 99.85% (E) 100.0%

41. A fair coin is tossed 8 times. Find the chance of getting exactly 7 heads.

    (A) 1/256 (B) 7/256 (C) 1/32 (D) 1/16 (E) 1/8 (F) 3/16

42. A fair coin is tossed 7 times. Find the chance of getting at most one head.

    (A) 1/128 (B) 7/128 (C) 1/16 (D) 1/8 (E) 3/16 (E) 15/16

43. A fair coin is tossed 102 times. Estimate the chance of getting exactly 53 heads.

    (A) 6.61% (B) 7.355% (C) 14.71% (D) 15.54% (E) 31.08%

44. A fair coin is tossed 67 times.

    (a) Find the precise chance of getting exactly 42 heads. Round to the nearest 0.01%.
    (A) 0.57% (B) 1.13% (C) 2.16% (D) 10.04% (E) 11.08%

    (b) Use the normal approximation to estimate the chance of getting exactly 42 heads.
    (A) 1.17% (B) 1.35% (C) 1.39% (D) 1.785% (E) 2.78%

For questions 45 and 46.

A box has nine tickets, numbered 2 through 10. Fifty draws are going to be made at random with replacement from the box.

45. The sum of the draws will be around ______, give or take _____. or so.
    (A) 275, 18.60 (B) 300, 2.58 (C) 300, 7.07 (D) 300, 18.26 (E) 300, 129

46. The chance that the sum of the draws will be within 30 of the expected number is about:
    (A) 8% (B) 20% (C) 90% (D) 99.07% (E) 100.0%

47. A fair coin is tossed 4 times.

The number of heads should come out to around ________ , give or take ________.

(A) 1/2, 1/2 (B) 2, 1/2 (C) 2, $\sqrt{2}/2$ (D) 2, 1 (E) 2, 2 (F) can't answer; sample size not large enough

48. A fair die is rolled 180 times.

The number of 4's rolled should come out to around ________, give or take ________.

(A) 30, $\sqrt{5}$ (B) 30, 1.86 (C) 30, 5 (D) 30, 7.5 (E) 60, 9

49. A fair die is rolled 1386 times.

The percent of 4's rolled should come out to around ________ percent, give or take ________ percent.

(A) $16\frac{2}{3}$, 0.373 (B) $16\frac{2}{3}$, 0.448 (C) $16\frac{2}{3}$, 1 (D) $16\frac{2}{3}$, 5.17 (E) 50, 18.61

50. A fair die is to be rolled 240 times.

There is an approximately 6.68% chance that the percentage of fours observed among the 240 rolls will be greater than what percent?

(This problem is a challenge problem.)

(A) 16.6667% (B) 18.280% (C) 19.072% (D) 20.275% (E) 45.7735%

51. A researcher spins a penny on a smooth surface to estimate the chances of it landing heads up. In 1,200 spins, the penny lands heads up 801 times. Assume that the chances are the same each time and the spins are independent.

A 95%-confidence interval for the chance of the coin landing heads is:

(A) 62.67 to 70.83% (B) 62.90 to 70.60% (C) 64.03 to 69.47% (D) 65.39 to 68.11% (E) 65.41 to 68.03%

52. A simple random sample of 500 women was taken from the whole country. The sample averaged 165.524 pounds with an SD of 27.364 pounds.

The average weight for all women in the U.S. is estimated at 165.524 pounds.

The 95%-confidence interval for the population average will be 165.524 pounds plus or minus ________ pounds.

(A) 0.66 (B) 1.224 (C) 2.4475 (D) 7.402 (E) 27.364

53. Two hundred draws are made at random with replacement from the box $\boxed{\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}}$. Which of the following five statements are true?

(a) The expected value for the average of the draws is exactly 2.

(b) The expected value for the average of the draws is around 2, give or take 0.05 or so.

(c) The average of the draws will be exactly 2.

(d) The average of the box is exactly 2.

(e) The average of the box is around 2, give or take 0.05 or so.

(A) all of them (B) a and c only (C) a and d only (D) b and d only (E) b, c, and e only

54. A truck is weighed 65 times on a truck scale. The 65 readings average 12,763 pounds, and the SD of the 65 readings is 38.7 pounds. (Assume the Gauss model.)

Which one of these statements is true?

(A) It is not possible to estimate the SD of the error box; all we have here is the SD of the sample. It is not enough.

(B) The standard error of the sample average—4.8 pounds—will estimate the SD of the error box.

(C) An estimate of the SD of the error box is 38.7 pounds.

(D) The SD of the error box is always assumed to be 0.

(E) The observed weighings are not just the errors from the error box; they are the results of adding the chance errors to the exact weight of the truck. So they cannot be used to estimate the chance error of the error box.

55. A coin was tossed 7500 times and got 3800 heads.

Set up an appropriate test of hypotheses. Then find P and decide if the coin is fair or gets too many heads.

(A) $P = 1.07\%$, not fair (B) $P = 2.14\%$, not fair (C) $P = 6\%$, fair
(D) $P = 12.5\%$, fair (E) $P = 25\%$, fair

56. A coin is tossed 8 times, and it lands heads 7 times. Is the chance of heads equal to 50%? Or are there too many heads for that?

Formulate the null and alternative hypotheses in terms of a box model.

Compute the observed significance level P and come to a conclusion.

Caution: do not use the normal approximation; the sample size is way too small.

(A) $P = 0.4\%$, not fair (B) $P = 3.5\%$, not fair (C) $P = 6\%$, fair (D) $P = 12\%$, fair (E) $P = 16\%$, fair

57. Each time that the Daily Number® is drawn, its first digit is a number from 0 to 9, chosen at random. To decide if all ten possibilities for the first digit are equally likely, a series of 400 test draws is run on a particular device. The results were as follows:
(0) - 34, (1) - 41, (2) - 32, (3) - 40, (4) - 26, (5) - 43, (6) - 44, (7) - 46, (8) - 45, (9) - 49.

Based on a test of hypotheses, find P and decide if the device is fair.

(A) 2%, not fair (B) 5%, not fair (C) 10%, fair (D) 25%, fair (E) 99%, fair

58. For each of the following statements, decide if it is true or false.

(a) The height of a block in a histogram is the population density in the inteval of the block.

(b) When the numbers in a class interval are truncated—such as with ages—the left end-point for the block should be a whole number.

(c) A typical number on a list will be off from the average of the list by an amount on the order of the SD.

(d) When a large random sample is taken from a large population, the sample average will be off from the population average by the SD of the box.

(e) The more you toss a fair coin, the closer the observed number of heads gets to the expected value.

(f) If the data don't follow the normal curve, you can't use the curve to get confidence intervals.