

Sample Final Examination

Math 125 *Kovitz* Spring 2008

On the final to get full credit you **must show your work**. No work, no credit.

The final will have ten questions, each worth 10 points with no penalty for wrong answers.

1. A large group of students at a certain university are required to take both the math and verbal SAT tests. Students who score high on the mathematics SAT also tend to score rather high on the verbal SAT. The correlation coefficient is $+0.4$. The scatter diagram is football-shaped.

- (a) Of the students who scored at the 96th percentile of math SAT scores:
 - i. just about half scored at or above the 96th percentile of verbal SAT scores.
 - ii. more than half scored at or above the 96th percentile of verbal SAT scores.
 - iii. less than half scored at or above the 96th percentile of verbal SAT scores.

Choose one of the options above, and explain.

- (b) Predict the percentile rank on the verbal SAT for a student whose percentile rank on the math SAT is 96%.
 - (c) Predict the percentile rank on the math SAT for a student whose percentile rank on the verbal SAT is 76%.
 - i. True or false: the answer to (c) could have been inferred from the answer to (b) without any calculation.
2. (a) Find the regression equation for the data set below. Then use that equation to predict y when $x = 1.5$.

x	y
3	4
3	2
1	1
3	5
4	5
4	7

- (b) A survey of a large group of men found that their average height was 67 inches with an SD of 4 inches, and their average weight was 152 pounds with an SD of 30 pounds, and their heights and weights followed the normal curve rather closely.
 - i. The slope of the regression line turned out to be 6 pounds per inch. Use the regression line (or the regression method for individuals) to predict the weight of a 72-inch man.
 - ii. A 72.4-inch man has what percentile rank in height? (Assume normal curve.)
 - iii. How tall is a man whose percentile rank in height is 40%? (Assume normal.)

3. Two draws will be made at random from a deck of cards. The unconditional probability of getting the king of hearts on the first draw is $1/52$. The unconditional probability of getting the king of hearts on the second draw is $1/52$.
- (a) The chance that the first draw turns out to be the king of hearts and the second draw turns out to be the king of hearts is $1/52 \times 1/52 = 1/2704$ when the draws are made _____ replacement, because then the events are _____.
 - (b) The chance that the first draw turns out to be the king of hearts and the second draw turns out to be the king of hearts is 0 when the draws are made _____ replacement, because then the events are _____.
 - (c) The chance that at least one of the two draws turns out to be the king of hearts is $1/52 + 1/52 = 1/26$ when the draws are made _____ replacement, because then the events are _____.
 - (d) The chance that at least one of the two draws turns out to be the king of hearts must be calculated as $1 - (51/52 \times 51/52) = 103/2704$ when the draws are made _____ replacement, because then the events are not _____.
 - (e) The chance that neither of the two draws turns out to be the king of hearts is $51/52 \times 51/52 = 2601/2704$ when the draws are made _____ replacement, because then the draws are _____.
 - (f) The chance that neither of the two draws turns out to be the king of hearts is $51/52 \times 50/51 = 50/52 = 25/26$ when the draws are made _____ replacement, because then the draws are not _____.

Fill in the blanks using one option from each pair below for each sentence.

(with, without) (independent, mutually exclusive)

4. A fair coin is tossed 40 times.
- (a) Find an expression for the exact probability of getting 20 heads. Evaluate that expression to get a percentage chance.
 - (b) Draw the middle rectangle of the probability histogram for the number of heads in 40 tosses and use the normal curve to estimate the answer to part (a). For that rectangle, label the exact endpoints, and indicate the length of the base.
5. A die is rolled 420 times. The total number of spots thrown is 1407.
- Find the expected value of the total of the spots on 420 rolls.
- Find the chance error in this total.
- What is this result (1407) in standard units?

6. In a certain town there are 30,000 registered voters, of whom 12,000 are Democrats. A survey organization is about to take a simple random sample of 1,000 registered voters.

- (a) The expected value for the percentage of Democrats in the sample is _____. The SE for the percentage of Democrats in the sample is _____.
- (b) The percentage of Democrats in the sample is likely to be around _____, give or take _____ or so.
- (c) Find the approximate chance that between 39.3% and 40.7% of the registered voters in the sample are Democrats? (Any correction factors may be ignored.)
- (d) Find the chance that exactly four of the first ten registered voters chosen for the sample are Democrats.

7. Twenty-five draws will be made at random with replacement from the box

1	2	3	4	5	6	7
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- (a) Find the chance (approximately) that the average of the draws will be within 2 of 4.
- (b) What percent of the tickets in the box are within 2 of 4? Why is this answer not the same as the answer to part (a)?
- (c) True or false:
 - i. 0.4 is the standard error for the average of the draws.
 - ii. 68% of the tickets in the box should be within 0.4 of the average of the box.
 - iii. Since the contents of the box do not follow the normal curve, the normal curve cannot be used to get chances for the average of the draws.
 - iv. There is about a 68% chance for the average of the 25 draws to be within 0.4 of the average of the box (in the range 3.6 to 4.4).

8. Laser altimeters can measure elevation to within a few inches, without bias, and with no trend or pattern to the measurements. As part of an experiment, 25 readings were made on the elevation of a mountain peak. These averaged out to 81,411 inches, and their SD was 30 inches. True or false; explain your answers briefly.

- (a) $81,411 \pm 12$ inches is a 95%-confidence interval for the elevation of the mountain peak.
- (b) There is about a 95% chance that the next reading will be in the range $81,411 \pm 12$ inches.
- (c) About 95% of the readings were in the range $81,411 \pm 12$ inches.

9. When convicts are released from prison, they have no money, and there is a high rate of “recidivism:” the released prisoners return to crime and are arrested again. Would providing income support to ex-convicts during the first months after their release from prison reduce recidivism? The Department of Labor ran a randomized controlled experiment to find out. The experiment was done of a selected group of convicts being released from certain prisons in Georgia. Income support was provided, like unemployment insurance. There was a control group which received no payment, and a treatment group.

Assume that the prisoners were randomized to treatment or control.

592 prisoners were assigned to the treatment group, and of them 48.3% were rearrested within a year of release. 154 were assigned to the control group, and of them 49.4% were rearrested within a year of release.

Did income support reduce recidivism? Answer yes or no, and explain briefly how you arrived at that conclusion based on a statistical test.

10. (a) One day, upon tossing a single die 600 times, I got:

108 ones, 93 twos, 114 threes, 120 fours, 93 fives, and 72 sixes.

Compute χ^2 and find P for this experiment.

- (b) Another day, upon tossing the same single die 300 times, I got:

57 ones, 57 twos, 51 threes, 39 fours, 54 fives, and 42 sixes.

Compute χ^2 and find P for this experiment.

- (c) Is the die biased, based on the combined evidence?

Clearly explain how you combined the results of the two days and justify your conclusion by comparing the P -value you obtained to the benchmark (5% or 1%) that you are using.