Solutions to Sample of Final Exam Questions

Math 125 *Kovitz* Fall 2010

1. The height of the block is 2% per thousand dollars. Each thousand-dollar interval between \$15,000 and \$25,000 contains about 2% of the families in the city. There are 7 of these thousand-dollar intervals between \$18,000 and \$25,000. The answer is $7 \times 2\% = 14\%$. About 14% of the families in the city had incomes between \$18,000 and \$25,000.

   The problem shows that with the density scale, the areas of the blocks come out in percent. The horizontal units—thousands of dollars—cancel. The area of the block shown is therefore:

   $$2\% \text{ per thousand dollars } \times 10 \text{ thousand dollars } = 20\%.$$

   And then, using a proportion, about 7/10 of the area of that block will be the answer since the base of a block for the incomes between \$15,000 and \$25,000 will be 7/10 of the base of the block pictured and we may assume that the density for the smaller block is approximately the same as the density for the block pictured. (There's no reason to assume otherwise.)

   Answer: about 14% of the families in the city had incomes between \$18,000 and \$25,000.

2. (a) The standard units for 72.4 is $\frac{72.4-67}{4} = 1.35$.
   The area under the normal curve from -1.35 to $+1.35$ is about 82%. Each tail is about $\frac{100-82}{2} = 9\%$. Add the lower tail to the middle area to get 91%, the percentile rank.

   (b) If the percentile rank is 40%, the left tail is 40%. The corresponding right tail will also be 40%. That leaves $100\% - 40\% - 40\% = 20\%$ in the middle. Look up that area to find the standard units of $z = 0.25$.
   In this case, the 40th percentile is below average. Use -0.25.
   So $-0.25 = \frac{x-67}{4}$, $-1 = x - 67$, and $x = 67 - 1 = 66$ inches.

   (c) The standard units for 62 and 66 inches are $\frac{62-67}{4} = -1.25$ and $\frac{66-67}{4} = -0.25$.
   Then take the area between -1.25 and -0.25 under the normal curve. This will be $\frac{1}{2}(78.87\%) - \frac{1}{2}(19.74\%) = 29.565\%$.

3. (a) with, independent.

   (b) with, mutually exclusive.

   (c) without, independent.

4. True.

5. (a) iii. Before you can calculate the expected value and standard error, you should set up the required box. A box for a percentage is always a 0–1 box. Here put one 1 and five 0's in the box, since we need to classify and count the results.

   (b) The expected value for the sample percentage is just equal to the percentage of 1's in the box we have set up: 16 2/3%.

   The standard error (give or take) for the sample percentage is just the SD of the counting box times 100% and divided by the square root of the number of draws. This gives

$$\frac{\sqrt{1/6 \times 5/6} \times 100\%}{\sqrt{1000}} = 1.1785\%.$$

   (c) The standard units for 19% is just $\frac{19\% - 16 2/3\%}{1.1785\%} = 1.98 \approx 2$.

   The area under the normal curve above 2 is $\frac{100\% - 95.45\%}{2} = 2.275\%$.

   (d) The give or take, the standard error, would be smaller if the sample size is made larger, since we divide by the square root of the sample size to calculate the give or take.

   (e)

$$\frac{\sqrt{1/6 \times 5/6} \times 100\%}{\sqrt{n}} = 0.25\%.$$

   Then

$$\sqrt{n} = 400 \times \sqrt{1/6 \times 5/6},$$

   and $n = 1/6 \times 5/6 \times 400^2 = 22,222$.

6. (a) Null hypothesis: the number of correct guesses is like the sum of 1,000 draws from a box with one ticket marked 1 and nine 0's.

   (b) $\sqrt{0.1 \times 0.9}$. The null hypothesis tells you what is in the box. Use it.

   (c) $z \approx (173 - 100)/9.5 \approx 7.7$, and P is tiny. SE sum $= 0.3 \times \sqrt{1000} \approx 9.5$.

   (d) Whatever it was, it wasn't chance variation.

7. $\chi^2 = 15.42$, 5 deg.fr., $P \approx 1\%$. Conclude that the die is biased at the 5% level.

   Calculations: all expected values are $600/6 = 100$.

$$\frac{(108-100)^2 + (93-100)^2 + (114-100)^2 + (120-100)^2 + (93-100)^2 + (72-100)^2}{100} = \frac{64 + 49 + 196 + 400 + 49 + 784}{100} = \frac{1542}{100} = 15.42.$$