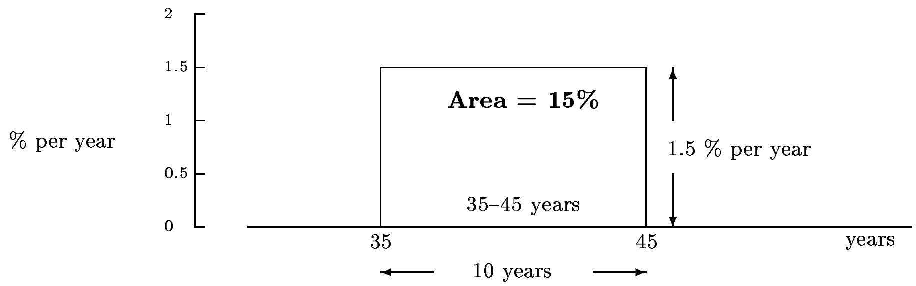
Answers to the Sample Final for Fall 2012

1.



2. (a) i. average = 4; deviations = −3,−1,0,1,3; SD = 2.
   ii. average = 12; deviations = −9,−3,0,3,9; SD = 6.

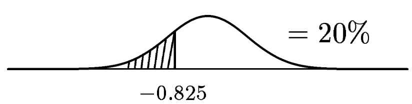
   (b) List (ii) is obtained from list (i) by multiplying each entry by 3. This multiplies the average by 3. It also multiplies the deviations from the average by a factor of 3, so, it multiplies the SD by a factor of 3. Multiplying each entry on a list by the same positive number just multiplies the SD by that number.

3. (a) True.
   (b) True. The correlation between y and x equals the correlation between x and y.
   (c) False. Association is not the same as causation.
   (d) False. Association is not the same as causation.

4. (a) 20% (b) 66% (c) 50%; the point of averages is always on the regression line.
   (d) 50%; predict average GPA.

Work for (a):

In standard units, his midterm grade was −1.65. The regression prediction for his final score is $0.5 \times 1.65 \approx -0.825$ in standard units.



This corresponds to a percentile rank of 20% or 21%, depending on how you rounded off. (It's close to 20.5%.)

Details of converting percentile to z: The lower tail is given to be 5%. The upper tail will also be 5%. That leaves 90% in the middle. Look up that percentage value on the table: 90%. It gives $z = -1.65$, because we want the value that is negative.

It is also possible to do the first step with a slightly different logic. Take the 5%, the given percentile rank and subtract it from 50% to get the area between $-z$ and 0 (45%). Then double the 45% to get the middle area (from $-z$ to z) for looking up on the table. Again look up 90% on the normal table and find z.

Details of converting z to a percentile, as required in the last step: $z = -0.825$ has slightly more than 59% in the middle. That is found by averaging 57.63% and 60.47% to get 59.05%. The two tails together will add to slightly less than 41%. Divide by 2 to get 20%, to the nearest percent, for the lower tail. And the lower tail is the percentile rank.

The percentile rank is predicted to be 20% (or 21%).

We are assuming that the percentile rank is to be a whole number. Otherwise the percentile rank would be about 20.5%.

Work for (b): The 80% below is 50% + 30%, so the area from 0 to z is 30%. For the area from $-z$ to z, just double the 30% to get 60%. For 60% in the middle, the positive value $z = 0.85$. Multiply 0.85 by 0.5 to get 0.425. The area from −0.425 to +0.425 is about 32.9%, averaging 31.08% amd 34.73%. Taking half of that area gives the area from 0 to z; it is 16.45%. To get the area below z add the area below 0, which is 50%, to the area from 0 to z.

Answer: 66%. (It's close to 66 1/2%, but we are using whole-numbered percentile ranks only.)

5. (a) 1/216
   (b) $1 - (5/6)^3 = 1 - 125/216 = 91/216 \approx 0.4213$, so about 42.1% or 42%
   (c) $(5/6)^3 = 125/216 \approx 0.5787$, so about 57.9% or 58%
   (d) $1 - 1/216 = 215/216 \approx 0.9954$, so about 99.5%. This result is the opposite of the result in (a).
   (e) 99.5%; you get at least one roll that is not an ace when you don't get all aces; so (d) and (e) are the same.

6. (a) Largest, 900; smallest, 100. (b) Chance $\approx$ 68%

7. The expected number of heads is 612.5; the SE is 17.5. You want the area of the rectangle over 626 in the probability histogram. That rectangle, being for counting numbers, has a base from 625.5 to 626.5.

   Find the standard units of the endpoints. Then the area between those standard units under the normal curve will give a valid approximation. The area under the normal curve between 0.75 and 0.8 is approximately 1.48%.

8. True. The result of the tosses, being modelled by the draws from a 0–1 box, will be off from the expected number of heads by an amount roughly comparable to the standard error.

   The observed number of heads = the expected value for the number of heads + chance error

9. (a) First use the SD of the sample to approximate the SD of the box.
   Then find the standard error for the average: $\frac{\text{SD of the box}}{\sqrt{\text{number of draws}}} = \frac{12}{\sqrt{25}} = \frac{12}{5} = 2.4.$
   The 95%-confidence interval is:
   sample avg $\pm 2 \times$ SE for the sample average = 299,789.2 $\pm$ 4.8 km per second.
   Answer: from 299,784.4 to 299,794.0 km per second.
   (b) No; it is 8.2 off from the average of the box. That's less than 1 SD from the average. Be careful: a single measurement—that's all we have here—is expected to be about 1 SD or so from the average.

10. (a) Tossing the coin is like drawing at random with replacement 10,000 times from a 0–1 box, with 0 = tails and 1 = heads. The fraction of 1's in the box is unknown.
    Null hypothesis: this fraction equals 1/2. Alternative: the fraction is bigger than 1/2.
    The number of heads is like the sum of the draws.
    (b) $z = 1.34$, $P \approx 9\%$.
    (c) The coin looks to be fair.

11. $\chi^2 = 1.0$, d.f. = 5, $95\% < P < 99\%$. The die looks fair. χ^2 was $\frac{1^2+1^2+0^2+2^2+2^2+0^2}{10}$.