

Sample of Final Examination Questions

Math 125 Spring 2017

On the final to get full credit you **must show your work**. No work, no credit.

The final has ten questions, each worth 10 points with no deduction for wrong answers.

1. A survey of a large group of men found that their average height was 67 inches with an SD of 4 inches.

- (a) A 72.4-inch man has what percentile rank in height? (Assume normal curve.)
- (b) How tall is a man whose percentile rank in height is 40%? (Assume normal.)

2. The observed ages of a group of interns at a local hospital were:

$\{33, 37, 32, 27, 28, 36, 38\}$.

- (a) Find the median of this list.
- (b) Find the mode of this list.
- (c) Find the average and the standard deviation of this list of ages. (Show your work.)
- (d) How many of the ages were within $\frac{1}{2}$ SD of the average age?
- (e) How many of the ages were within 1.75 SDs of the average age?
- (f) Using the results of part (a), convert the observed value of 28 to standard units.

3. For the data set below:

x	y	
3	4	(a) find r , the correlation coefficient.
3	2	(b) based on these calculations, predict y when $x = 1.5$.
1	1	
3	5	(c) find the root-mean-square error of the regression line
4	5	that is used to predict y from x .
4	7	(d) find the slope of that regression line.

4. Six cards are drawn from a standard deck of cards. In each suit, the numbered cards are the nine cards from 2 to 10, inclusive. The Ace shall not be considered a numbered card for this problem. Find the probability (rounded to the nearest .01%) that

- (a) at least one of the cards drawn is a king, if they are drawn
 - i. without replacement.
 - ii. with replacement.
- (b) all six of the cards drawn are numbered cards, if they are drawn
 - i. without replacement.
 - ii. with replacement.

5. A certain town has 25,000 families. These families own 2.1 cars, on the average, with an SD of 0.80. And 10% of them have no cars at all.

If one draws a random sample of six families from the 25,000, what is the chance that exactly two of the six sample families will not own cars? (Assume, for simplicity of calculation, that the draws are made with replacement).

6. A fair coin is tossed 40 times.

- (a) Find an expression for the exact probability of getting 20 heads. Evaluate that expression to get a percentage chance.

- (b) Draw the middle rectangle of the probability histogram for the number of heads in 40 tosses and use the normal curve to estimate the answer to part (a). For that rectangle, label the exact endpoints, and indicate the length of the base.
7. A researcher takes a simple random sample of 900 students from the 13,000 students at UMass. On the average, there are 0.52 cats per sample student, and the SD is 0.152. Say whether each of the following statements is true or false, and explain.
- (a) The 0.52 is 0.152 or so off the average number of cats per student in the whole school.
 - (b) A 95%-confidence interval for the average number of cats per student in the sample is 0.216 to 0.824.
 - (c) A 95%-confidence interval for the average number of cats per student in the whole school is 0.51 to 0.53.
 - (d) 95% of the students at UMass have between 0.216 and 0.824 cats.
 - (e) The 95%-confidence level is about right because the number of cats follows the normal curve.
 - (f) The 95%-confidence level is about right because, with 900 draws from the box, the probability histogram for the average number of cats in the sample follows the normal curve.
8. A survey organization takes a simple random sample of 625 households from a city of 80,000 households. On the average, there are 2.30 persons per sample household, and the SD is 1.75. That household size will not follow the normal curve is fairly clear considering that household sizes of 10 are possible but household sizes of -6 are not. If the histogram for the population is not symmetric about the population average, it cannot follow the normal curve.

Say whether each of the following statements is true or false, and explain.

- (a) Since the population does not follow the normal curve, you cannot use the curve to get confidence intervals.
 - (b) A 95%-confidence interval for the average household size in the city is 2.16 to 2.44.
 - (c) There is a 95% chance that the average household size in the entire city lies in this above-mentioned confidence interval.
9. Laser altimeters can measure elevation to within a few inches, without bias, and with no trend or pattern to the measurements. As part of an experiment, 25 readings were made on the elevation of a mountain peak. These averaged out to 81,411 inches, and their SD was 30 inches.
- (a) If a 26th reading were made, you would expect to get 81,411 inches give or take how much?
- (Say whether each of (b-e) is true or false and explain your answers briefly.)
- (b) The average of all 25 readings is off 81,411 inches by 6 inches or so.
 - (c) $81,411 \pm 12$ inches is a 95%-confidence interval for the elevation of the mountain peak.
 - (d) There is about a 95% chance that the next reading will be in the range $81,411 \pm 12$ inches.
 - (e) About 95% of the readings were in the range $81,411 \pm 12$ inches.

10. True or false: with a well-designed sample survey, the sample average is very likely to equal the population average. Explain.
11. (a) One day, upon tossing a single die 600 times, I got:

108 ones, 93 twos, 114 threes, 120 fours, 93 fives, and 72 sixes.

Compute χ^2 and find P for this experiment. Is the die biased, based on those 600 tosses?

Justify your conclusion by comparing the P -value you obtained to the benchmark (5% or 1%) that you are using.

- (b) After concluding as I did, I gave it some further thought. The number of times that resulted in a three seemed to be somewhat close to what is expected.

Does the fact that 114 threes came out in the 600 tosses indicate that—at least as far as the threes are concerned—the die is fair? (Clearly describe the statistical test you decide to use, state the hypotheses, and show the computations you performed that justify that conclusion.)

- i. Into what trap have I fallen by deciding to test the 3's? Does this make the conclusion from the test of the 3's somewhat suspect? Explain.

12. When convicts are released from prison, they have no money, and there is a high rate of “recidivism:” the released prisoners return to crime and are arrested again. Would providing income support to ex-convicts during the first months after their release from prison reduce recidivism? The Department of Labor ran a randomized controlled experiment to find out. The experiment was done of a selected group of convicts being released from certain prisons in Georgia. Income support was provided, like unemployment insurance. There was a control group which received no payment, and a treatment group.

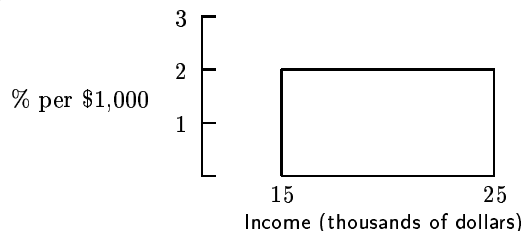
Assume that the prisoners were randomized to treatment or control.

592 prisoners were assigned to the treatment group, and of them 48.3% were rearrested within a year of release. 154 were assigned to the control group, and of them 49.4% were rearrested within a year of release.

Did income support reduce recidivism? Answer yes or no, and explain briefly how you arrived at that conclusion based on a statistical test.

A1 and A2 are substitute problems. On the final, use either or both to replace previous problems, if you wish.

- A1. The sketch below shows one block of the family-income histogram for a certain city. About what percent of the families in the city had incomes between \$19,000 and \$25,000?



- A2. One hundred draws are made at random with replacement from the box

1	2	3	4	5	6	7
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The sum of the draws is 431. The expected value for the sum of the draws is _____, the observed value is _____, the chance error is _____, and the standard error is _____.

Fill in the blanks, and explain briefly.

Is the observed result in line with what was expected?