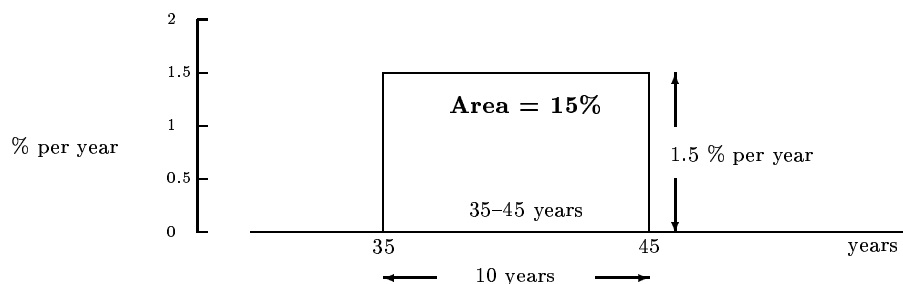


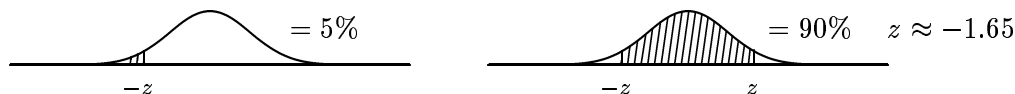
Answers to the Sample Final for Fall 2018

1.



2. (a) i. average = 4; deviations = $-3, -1, 0, 1, 3$; SD = 2.
 ii. average = 12; deviations = $-9, -3, 0, 3, 9$; SD = 6.
- (b) List (ii) is obtained from list (i) by multiplying each entry by 3. This multiplies the average by 3. It also multiplies the deviations from the average by a factor of 3, so, it multiplies the SD by a factor of 3. Multiplying each entry on a list by the same positive number just multiplies the SD by that number.
3. (a) True.
 (b) True. The correlation between y and x equals the correlation between x and y .
 (c) False. Association is not the same as causation.
 (d) False. Association is not the same as causation.
4. (a) 20% (b) 66% (c) 50%; the point of averages is always on the regression line.
 (d) 50%; predict average GPA.

Work for (a):



In standard units, his midterm grade was -1.65 . The regression prediction for his final score is $0.5 \times 1.65 \approx -0.825$ in standard units.



This corresponds to a percentile rank of 20% or 21%, depending on how you rounded off. (It's close to 20.5%.)

Details of converting percentile to z : The lower tail is given to be 5%. The upper tail will also be 5%. That leaves 90% in the middle. Look up that percentage value on the table: 90%. It gives $z = -1.65$, because we want the value that is negative.

It is also possible to do the first step with a slightly different logic. Take the 5%, the given percentile rank and subtract it from 50% to get the area between $-z$ and 0 (45%). Then double the 45% to get the middle area (from $-z$ to z) for looking up on the table. Again look up 90% on the normal table and find z .

Details of converting z to a percentile, as required in the last step: $z = -0.825$ has slightly more than 59% in the middle. That is found by averaging 57.63% and 60.47% to get 59.05%. The two tails together will add to slightly less than 41%. Divide by 2 to get 20%, to the nearest percent, for the lower tail. And the lower tail is the percentile rank.

The percentile rank is predicted to be 20% (or 21%).

We are assuming that the percentile rank is to be a whole number. Otherwise the percentile rank would be about 20.5%.

Work for (b): The 80% below is $50\% + 30\%$, so the area from 0 to z is 30%. For the area from $-z$ to z , just double the 30% to get 60%. For 60% in the middle, the positive value $z = 0.85$. Multiply 0.85 by 0.5 to get 0.425. The area from -0.425 to $+0.425$ is about 32.9%, averaging 31.08% and 34.73%. Taking half of that area gives the area from 0 to z ; it is 16.45%. To get the area below z add the area below 0, which is 50%, to the area from 0 to z .

Answer: 66%. (It's close to $66 \frac{1}{2}\%$, but we are using whole-numbered percentile ranks only.)

5. (a) $1/216$
 (b) $1 - (5/6)^3 = 1 - 125/216 = 91/216 \approx 0.4213$, so about 42.1% or 42%
 (c) $(5/6)^3 = 125/216 \approx 0.5787$, so about 57.9% or 58%
 (d) $1 - 1/216 = 215/216 \approx 0.9954$, so about 99.5%. This result is the opposite of the result in (a).
 (e) 99.5%; you get at least one roll that is not an ace when you don't get all aces; so (d) and (e) are the same.
6. (a) Largest, 900; smallest, 100. (b) Chance $\approx 68\%$
7. The expected number of heads is 612.5; the SE is 17.5. You want the area of the rectangle over 626 in the probability histogram. That rectangle, being for counting numbers, has a base from 625.5 to 626.5. Find the standard units of the endpoints. Then the area between those standard units under the normal curve will give a valid approximation. The area under the normal curve between 0.75 and 0.8 is approximately 1.48%.
8. True. The result of the tosses, being modelled by the draws from a 0–1 box, will be off from the expected number of heads by an amount roughly comparable to the standard error.
- The observed number of heads = the expected value for the number of heads + chance error
9. (a) First use the SD of the sample to approximate the SD of the box.
 Then find the standard error for the average: $\frac{\text{SD of the box}}{\sqrt{\text{number of draws}}} = \frac{12}{\sqrt{25}} = \frac{12}{5} = 2.4$.
 The 95%-confidence interval is:
 sample avg $\pm 2 \times \text{SE}$ for the sample average = $299,789.2 \pm 4.8$ km per second.
 Answer: from 299,784.4 to 299,794.0 km per second.
- (b) No; it is 8.2 off from the average of the box. That's less than 1 SD from the average. Be careful: a single measurement—that's all we have here—is expected to be about 1 SD or so from the average.
10. (a) Tossing the coin is like drawing at random with replacement 10,000 times from a 0–1 box, with 0 = tails and 1 = heads. The fraction of 1's in the box is unknown.
 Null hypothesis: this fraction equals $1/2$. Alternative: the fraction is bigger than $1/2$.
 The number of heads is like the sum of the draws.
- (b) $z = 1.34$, $P \approx 9\%$.
- (c) The coin looks to be fair.
11. $\chi^2 = 1.0$, d.f. = 5, $95\% < P < 99\%$. The die looks fair. χ^2 was $\frac{1^2+1^2+0^2+2^2+2^2+0^2}{10}$.