

## Answers to Sample Final Examination

Math 125 *Kovitz* Spring 2021

1. If 74% of the scores are above 450, 450 must be below average (on the normal curve 50% of the scores are above average.) Since 74% of the area under the normal curve is to the right of 450 points in standard units, we conclude that the area to the left of that value, called  $-z$ , is 26%. By symmetry, the area to the right of  $+z$  is 26%. That means that the area between  $-z$  and  $z$  is  $100\% - 26\% - 26\% = 48\%$ . Thus  $z \approx 0.65$ , and the value for 450 points in standard units is  $\approx -0.65$ .

Since 450 in standard units equals  $-0.65$ , 450 is 0.65 SDs to the left of the average score. 0.65 SDs are equal to  $0.65 \times 80$  or 52 points. Because the score of 450 is 52 points below average, the average is  $450 + 52 = 502$  points.

By formula, we could have shown that  $-0.65 = z = (450 - \text{average})/80$  and solved for the average, obtaining 502.

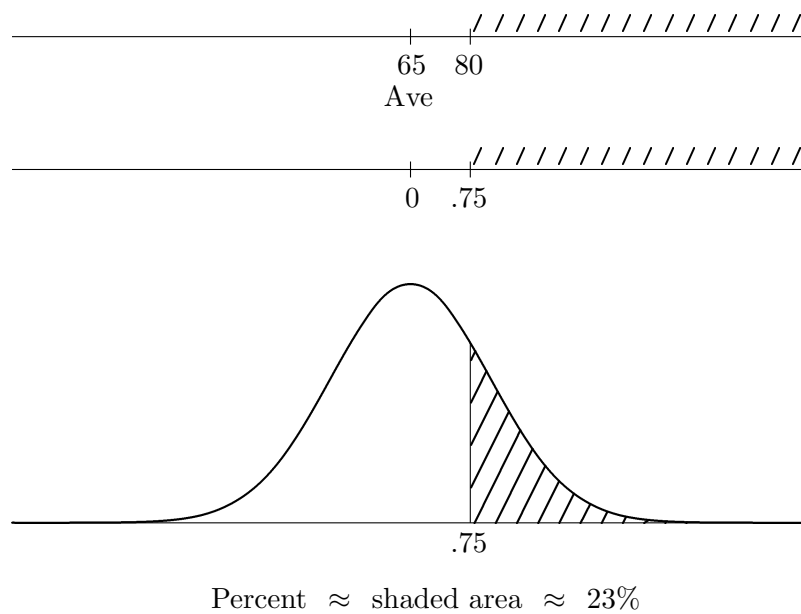
2. The averages are 7 and 19, respectively; the SDs are 2 and 4, respectively. The average product of the values in standard units is then

$$\frac{(1.5 \times -1) + (0.5 \times 1.5) + (0 \times 0) + (-1 \times -1.5) + (-0.5 \times -0.5) + (1 \times 1) + (-1.5 \times 0.5)}{7}$$

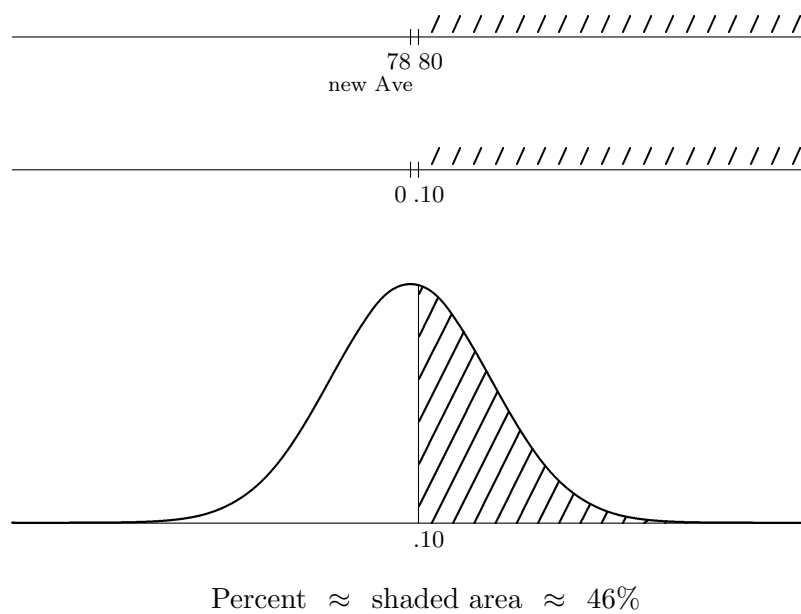
$$= \frac{1.25}{7} \approx 0.18.$$

$$r = 0.18.$$

3. (a)



- (b) new average final score =  $65 + (1.33 \times 0.50 \times 20) \approx 78$  (since 80 on the midterm in standard units is  $(80 - 60)/15$ ),  
 new SD =  $\sqrt{1 - 0.5^2} \times 20 = \sqrt{0.75} \times 20 = 0.866 \times 20 \approx 17$ ,  
 $(80 - 78)/17 \approx 0.10$



4. The average of all men in the sample that were 66 inches tall is estimated to be 143 pounds. Work: 66 inches is 4 inches, or  $4/3 \approx 1.33$  SDs below average. The estimated weight is below the average weight by  $r \times 1.33 = 0.47 \times 1.33 \approx 0.625$  SDs. This is  $0.625 \times 30 \approx 19$  pounds. This man is a little lighter than the estimated average of all the men, so he is a little lighter than the average of all the other men.
5. It could be that the money made the players fat and lazy, but this looks a lot like the regression effect.
6. This is like 400 draws from the box  $\left[ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 1 & 1 & 1 \\ \hline \end{array} \right]$ . The average of this box is 0.6 and the SD is  $(1 - 0)\sqrt{0.6 \times 0.4} \approx 0.49$ . The expected value for the sum is  $400 \times 0.6 = 240$ ; the SE for the sum is  $\sqrt{400} \times 0.49 = 9.8$ .
  - (a) Since the number of draws is large, the normal curve may be used to estimate these binomial chances. We must find the endpoints of the relevant rectangles of the actual histogram. For less than 220 males, the endpoint is 219.5. The value in standard units is  $(219.5 - 240)/9.8 = -20.5/9.8 \approx -2.10$ . The area under the normal curve to the left of  $-2.10$  is  $(100\% - 96.43\%)/2 \approx 2\%$ .
  - (b) The endpoints of the rectangle of the probability histogram are 219.5 and 220.5.  
 The values in standard units are  $(219.5 - 240)/9.8 = -20.5/9.8 \approx -2.10$  and  $(220.5 - 240)/9.8 = -19.5/9.8 \approx -2.00$ .  
 The area under the normal curve between  $-2.10$  and  $-2.00$  is  $96.43\%/2 - 95.45\%/2 \approx 0.5$  of 1%.
7.
  - (a) True.
  - (b) False: the sample percentage is known, and in the interval.
  - (c) False. The population percentage either is in the interval or is not in the interval—there is no chance involved. See page 351 in the text.
  - (d) True. Two SEs either way—3.2 percentage points either way—constitutes a confidence interval. About 95.45% of the 2,000 intervals would cover the population percentage—that is, would be off the actual percentage of Democrats in the large population by 3.2 percentage points of less. That leaves 4.55% of the 2,000 samples, meaning 91 samples, that would be off by more than 3.2 percentage points.
8.
  - (a) False. A and B can both happen together, with chance  $1/3 \times 1/10$ . So they aren't mutually exclusive.
  - (b) True. If A happens, the chance of B drops to 0. That's an extreme form of dependence.

9. Option (ii) is right. In section 20.4 we saw that accuracy is determined not by the size of the population, but only by the size of the samples (see page 367). Since the Los Angeles sample is four times as large, its accuracy will be twice as big (see page 360). If the SE is halved, one says that the accuracy is doubled.
10. (a) 300,007 (the average); 2 (the SE for the average)  
 (b) True: each number on a list is off the average of the list by an SD or so.  
 (c) False: the average is 300,007 exactly.  
 (d) False: 2 is the SE, not the SD.  
 (e) True: the interval is “average  $\pm$  2 SEs.”  
 (f) False: the average of the 25 measurements is 300,007 exactly.
11. (a) True.  
 (b) False. There is no such thing as a 95%-confidence interval for the *sample average*; you *know* the sample average. It’s the population average that you have to worry about.  
 (c) True.  
 (d) False, This confuses the SD with the SE. And it’s ridiculous in the first place, because a household must have a whole number of persons (1, or 2, or 3, and so forth). The range 2.16 to 2.44 is impossible for any particular household, let alone 95% of them; although this range is fine for the average of all the households.  
 (e) False. For instance, if household size followed the normal curve, there would be many households with a negative number of occupants; we’re not ready for that.  
 (f) False. Household size does not follow the normal curve, but you can use the normal curve to approximate the probability histogram for the sample average (pages 382–383).
12. (a) 16.0 ounces  $\pm$  0.32 ounces or so. (b) 98.76%. These are sums.
13. 9480 seconds  $\pm$  24.6 seconds or so. These are sums.
14. (a) 10,000. (b) A zero or a one—10% ones.  
 (c) False—it’s  $\sqrt{0.1 \times 0.9} = 0.3$ . (d) True.  
 (e) 68%—the SE for the percentage is 1%.  
 (f) 2%—use the binomial formula. (The sample is so small relative to the population that the chances are practically the same as if the draws were with replacement; hence the draws are nearly independent. The normal approximation cannot be used since the sample size  $n$  is so small.) Here:  $n = 7$ ,  $k = 3$ ,  $n - k = 4$ ,  $p = .1$ , and  $1 - p = .9$ .  
 (g) It is not possible since we do not know that the distribution follows the normal curve. We would need more information to calculate this chance.

15. (a) 10%—use the binomial formula. (Same comment as in 3 (f).)  
 (b) 82%.
16. 40,000.
17. 15.
18. (a)  $\chi^2 = 13.2$ , 5 deg.fr.,  $1\% < P < 5\%$ . (b)  $\chi^2 = 10$ , 5 deg.fr.,  
 $P \approx 7.5\%$ .  
 (c) Pooled  $\chi^2 \approx 13.2 + 10 = 23.2$ ,  $5 + 5 = 10$  deg.fr.,  $P \approx 1\%$ ;  
 conclude that the die is biased.
19. According to the Gauss model, each of the 100 measurements equals the exact weight, plus bias, plus a draw from the error box. The null hypothesis says that the bias is zero; the alternative hypothesis says that there is some bias. The SD of the error box is estimated as 40, from the bootstrap. (The errors belong to the rebuilt scale, so the old SD of 47 micrograms is irrelevant.) The SE for the average is  $40/\sqrt{100} = 4$  micrograms,  $z = -0.75$ , The area to the left of  $-0.75$  for the normal curve is about 23%. The null looks good.
20. The SD of the box can be estimated as 600, so the SE for the average of 400 draws is estimated as 30. If the average of the box is 1000, then the average of the draws is 2.82 SEs above its expected value. This isn't plausible.
21.  $z \approx 2.4$  and  $P < 1\%$ . The null hypothesis does not look good.
22. (a) The data are like 400 draws from a box, with one ticket in the box for each six-year-old in Maine, showing his score. The null hypothesis is that the average of the box equals 50, the alternative is that the average is bigger than 50.  
 (b) The SD of the box is estimated as 12 (no reason it should be the same as for the nation), the SE for the average of 400 draws is 0.6, so  

$$z = (51.4 - 50)/0.6 \approx 2.35 \text{ and } P < 1\%.$$
  
 (c) There is good evidence that the average for Maine is bigger than 50 (that is, there is good evidence that it was not a chance variation).

23. (a) The data is like the result of drawing 225 times at random (without replacement) from a box of tickets. There is one ticket in the box for each school, showing its enrollment. The null hypothesis is that the average of the box is 3,700; the alternative, that the average of the box is less than 3,700.
- (b) The SD of the box is unknown, but can be estimated by the SD of the sample, as 6,000. So the SE for the average of 225 draws is 400, and

$$z = \frac{3,500 - 3,700}{400} \approx -0.5, P \approx \text{[graph of normal distribution with area to the left of -0.5 shaded]} \approx 31\%.$$

- (c) The difference could easily be a chance variation.
24. Aside from any ethical or legal reasons why you should not bring a bomb on board, any thought that this will make it less likely that someone else will bring a bomb on board is a fallacy.

If you do not bring a bomb on board, the chances of someone bringing a bomb on board are 1 in 1000 as stated. However, if you do bring a bomb on board the chances of two people bringing a bomb on board are *not* 1 in 1,000,000, but rather 1 in 1000. We multiply the chances of you bringing a bomb on board (in the case where you bring one), 1 chance in 1 (a sure thing), by the chances of someone else bringing a bomb on board, 1 chance in 1000, and obtain 1 chance in 1000, just as before. The chances of 1 in 1,000,000 were computed for two independent (which we still have here) and random (which is not true for both) events. The event of your bringing a bomb on board, in this case where you bring one, is no longer random.

This fallacy is similar to the fallacy of saying that if a fair coin comes up heads 10 times in a row, it is not likely to come up heads on the next toss, because the chances of its coming up heads eleven times in a row are 1 in 2048. The fallacy is that the chances *were* 1 in 2048 in the beginning; once the first ten tosses come up heads, the chances that the eleventh toss (and thus, in that case, all eleven tosses) comes up heads is exactly 1 in 2, as on all other tosses of a fair coin.