Solutions to Sample of Final Exam Problems

(Final on December 20)
Math 125, Fall 2024

Multiple Choice. $3\frac{1}{2}$ points for each correct response, no points deducted for a wrong answer

1. The percentage for the block with base \$75,000 to \$90,000 is the producet of the width of the base and the height of the block.

   Here: (\$90,000-\$75,000) $\times$ 2% per \$1,000 = \$15,000 $\times$ 2%/\$1,000 = 30%. Answer: (C)

   For a rectangle: AREA = (LENGTH times WIDTH) or (BASE times HEIGHT).

2. (a) The average is $\frac{2+3+4+5+6+7+8+9+10}{6} = \frac{54}{9} = 6$.
   The SD is $\sqrt{\frac{16+9++4+1++0+1+4+9+16}{9}} = \sqrt{\frac{60}{9}} = \sqrt{\frac{20}{3}} = \sqrt{60}/3 = 2.582$.
   The standard units equals $\frac{\text{obs-avg}}{\text{SD}} = \frac{10-6}{2.582} = \frac{4}{2.582} = 1.549$.
   The answer is: (C) 1.549.

   Alt. Method to get SD (top of pg. 74): avg of squares is (4+9+16+25+36+49+64+81+100)/9=384/9=128/3.
   Then the square of the average is $6^2 = 36$. SD $= \sqrt{36 - (128/3)} = \sqrt{20/3} = 2.582$.

   (b) Within 1.25 SDs of the average means within $1.25 \times 2.882 = 3.23$ or less away from 6. The limits are 2.77 and 9.23. In that range are 7 of the entries, so the percentage is $7/9 \times 100\% = 78\%$.
   The answer is: (D) 78%.

   (c) For the normal curve, the percent within 1.25 SDs of the average means the area under the curve between $z = -1.25$ and $z = +1.25$ (it starts at 1.25 SDs below 0, which is standard units of -1.25, and continues to 1.25 SDs above 0, which is standard units of $+1.25$).
   The required area from the Normal Table is 78.87%, so the answer is: (D) 79%.

3. Since fifty percent scored above the average, to endup with 74% above 450 points, 24% must have scored between 450 points and the average. Let $-z$ be the standard units for 450 points.

   Between $-z$ and 0 is 24% of the area under the normal curve.

   That means between $-z$ and z is 48% of the area. Conclude that $-z$ is -0.65.

   If 450 points has standard units equal to -0.65, it must be 0.65 SDs below the average.

   (See the box on page 79 in the text.)

   0.65 SDs $= 0.65 \times 80$ points = 52 points below the average. So 450 pts. is 52 pts. below the average.

   The average is 450 points + 52 points = 502 points. Answwer: (D)

4. This problem is very similar to Example 10 on pages 90 and 91.

   First find the standard units for the 93rd percentile.

   The definition of the 93rd percentile requires a height with 93% of the heights below it. That means the z (standard units) for that height will be the z that has 7% of the normal curve above it. By symmetry, 7% of the normal curve will be below $-z$.

   The area between z and z must be $100\% - 7\% - 7\% = 86\%$. From the table, we see that $z \approx +1.475$, taking a value halfway betwen 1.45 and 1.50 as the value.

   A woman has to be about 1.475 SDs above average to be in the 93rd percentile of the height distribution. Translated back to inches, this height is above average by $1.475 \times 3 = 4.425$ inches.

   Answer: (D). The 93rd percentile of the height distribution is $63.5 + 4.425 = 67.925$ inches.

5. (C) 43%

   The endpoints in standard units are:

   $\frac{64-65}{3.32} = \frac{-1}{3.32} = -0.30$ and

   $\frac{68-65}{3.32} = \frac{3}{3.32} = 0.90$.

   The area from -0.30 to 0.90 is broken up into two parts:

   The area from -0.30 to 0 (one half of the area from -0.30 to $+0.30$), which comes out to 11.79%, and

   the area from 0 to 0.90 (one half of the area from -0.90 to $+0.90$), which comes out to about 31.595%.

   Add the two areas to get 11.79% + 31.595%, or about 43%.

6. (C) 0.375

   First find the average x and the average y:

   avg $x = \frac{6+8+2+10+14}{5} = \frac{40}{5} = 8$ and avg $y = \frac{3+11+12+9+15}{5} = \frac{50}{5} = 10$.

   The find SD of x and SD of y:

   SD of $x = \sqrt{\frac{(-2)^2+0^2+(-6)^2+2^2+6^2}{5}} = \sqrt{\frac{80}{5}} = 4$. Same for SD of y; it equals 4.

   Now convert each of the ten numbers in the original table to standard units, and put the ten results in a new table.

   Proceed like this: $\frac{6-8}{4} = \frac{-2}{4} =$ -0.5.

   The ten results are put in the new table and each row is multiplied across to get the products.

x std. units	y std. units	product
-0.5	-1.5	0.875
0	0.25	0
-1.5	0.5	-0.75
0.5	-0.25	-0.125
1.5	1.25	1.875

   Then get the average of the 5 products. They add to 1.875; divide by 5: 0.375.

   That is r. The correlation coefficient, r, for the above data set is: (C) 0.375

   Alt. Method to get SD for x (top of pg. 74): avg of squares is (36+64+4+100+196)/5=400/5=80.

   Then the square of the average is $8^2 = 64$. SD $= \sqrt{80-64} = \sqrt{16} = 4$.

   Alt. Method to get SD for y (top of pg. 74): avg of squares is (9+121+144+81+225)/5=580/5=116.

   Then the square of the average is $10^2 = 100$. SD $= \sqrt{116-100} = \sqrt{16} = 4$.

   Alt. Method to get r (pg. 134, Techincal Note): avg of products is (18+88+24+90+210)/5=430/5=86.

   Then the product of the averages is $8 \times 10 = 80$. $r = (86-80)/(\text{SD of } x \times \text{SD of } y) = 6/(16) = 0.375$.

7. Use the simple formula:

   predicted y = average y + [(x in standard units) $\times$ r $\times$ (the SD of y)].

   The weight of 225 pounds is $\frac{225-175}{45} = \frac{50}{45} = 10/9$ in standard units.

   The formula yields: $69 + [10/9 \times 0.42 \times 3] = 69 + 1.4 = 70.4$ inches.

   Alternative Solution without the Formula: (See the top half of page 160 in the text.)

   The height of 225 pounds is 50 pounds above 175 pounds, the average. That is $50/45 = 10/9$ SDs above average.

   These men who weigh 225 pounds should be r times 10/9 SDs above average in height. So they are $0.42 \times 10/9 = 0.46667$ SDs above average in height.

   That's 0.46667×3 inches = 1.4 inches. So their average is around $69 + 1.4 = 70.4$ inches. Use that estimated average for the prediction. (See page 165 to top of page 166 in the text.)

   Answer: (D) 70.4 inches

8. Answer: (B) 0.42.

   See the text on page 141 up to the first paragraph on page 142.

   Conversion from inches to centimeters is just a change of scale which does not affect the correlation (r).

   See the third bullet on the box on page 143. Change of scale is just multiplcation by the same positive number, the conversion factor.

9. Answer: (A) 72 inches.

   His height stayed the same.

   Correlation measures association. But association is not the same as causation. (See page 150.)

10. Answer: (C) 2.72257 inches

    The formula is $\sqrt{1-r^2}$ $\times$ the SD of the variable being predicted. (Top box on page 186.)

    Here: $\sqrt{1-r^2} = \sqrt{1-0.42^2} = \sqrt{1-0.1764} = \sqrt{0.8236} = 0.907524$.

    Multiply 0.907524 by 3 inches to get 2.72257 inches, answer (C).

11. Answer: (A) $y = 0.028x + 64.1$

    The slope of the regression line is: $\frac{r \times \text{SD of } y}{\text{SD of } x}$.

    Here, the slope is $\frac{0.42 \times 3}{45} = 0.028$.

    Next write the equation as $y = 0.028x + b$, plug in a known point (x, y) on the line, and solve for b. The point of averages is always on the regression line. That would be a good choice here. It is $x = 175$, $y = 69$.

    $69 = 0.028(175) + b$, $69 = 4.9 + b$, $b = 69 - 4.9 = 64.1$. The line is $y = 0.028x + 64.1$, answer (A).

12. Answer: (A) $0.028(225) + 64.1 = 70.4$.

    This is the same as the answer to problem 8, as it should be.

13. Answer: (C) 37.8 pounds

    Here we are predicting weight from height so we need the slope of the regression line that predicts weight from height. That will be when x is the height variuable and y is the weight variable.

    The slope of the regression line is then: $\frac{r\times \text{SD of } y}{\text{SD of } x}$.

    Here, the slope is $\frac{0.42\times 45}{3} = 6.3$.

    Next, use the equation: slope = (change in y)/(change in x).

    6.3 = (change in weight)/6

    6.3×6 = change in weight.

    The change in weight will be 37.8 pounds. That is from the regresssion line.

14. Estimate the average weight of all 66-inch-tall men:

    estimated average weight = 162 pounds + $((-4)/3 \times 0.47 \times 30$ pounds) =
    162 pounds + $(-1.333333 \times 0.47 \times 30$ pounds) = 162 pounds − 18.8 pounds = 143.2 pounds.

    Since the other men averaged 143.2 pounds, he was a little light. Answer: (A)

15. An average man who is one SD above average in height will generally be about r SDs above average in weight.

    (See the bottom box on page 160.)

    His 1 SD above average is larger than the 0.47SDs above average for all men of his height.

    Answer: (A) more.

16. (D) 65%

    Do not add up four chances of hitting the bullseye. The outcomes are not mutually exclusive; he could hit the bullseye more than once. (See the box on page 242 in the text.)

    Instead look to the opposite outcome. The opposite of hitting the bullseye at least once is not hitting it at all in the four throws. That outcome is "he missed the bullseye on all four throws."

    Since the results are independent, just multiply together four factors, each of them the unconditional chance of missing (0.77). (See the box on page 232 in the text.)

    We have just found the chance of the opposite of "at least one of the four throws results in a bullseys" to be equal to $(0.77)^4 \approx 0.35 = 35\%$. (Now refer to the second box on page 223 of the text.)

    The chance originally requested (of hitting the bullseye at least once) is

    100% − 35% = 65%, answer (D).

17. (F) 42%

    This is a binomial probability. The clue is the word "exactly" in the question. Find n, k, and p.

    $n = 4$, $k = 1$, $n - k = 3$, $p = 0.23$, and $1 - p = 0.77$.

    This is hitting the bullseye once, so $k = 1$ and $p = 0.23$, the chance of hitting the bullseye on each throw.

    The binomial formula is: The probability $= \frac{n!}{k!\times(n-k)!}p^k(1-p)^{n-k}$. (page 259)

    Here $\frac{4!}{1!\times 3!}(0.23)^1(0.77)^3 \approx 0.42 = 42\%$, answer (F).

18. (B) 2/17.

    Without replacement, the draws are dependent. Use two applications of the Multiplication Rule found in the box on page 229 of the text.

    Calculating the chance using conditional probabilities:

    $$\frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{1}{2} \times \frac{25{\times}2}{17{\times}3} \times \frac{3{\times}2{\times}2{\times}2}{25{\times}2}.$$

    After cancellation: 2/17.

    Answer: (B).

19. The chance of getting 10 sixes is $(1/6)^{10}$.

    The chance of its opposite is found by subtracting that chance from 1.

    (See the second box on page 223, using the decimal instead of the percent.)

    The chance will be: $1 - (1/6)^{10}$), Answer: (D).

20. Answer: (F) 31/36.

    - One idea is to apply the de Méré method, detailed in the box on page 250.
    - Another method is possible. It is presented in footnote 2 to Chapter 14 on page A-16.
    - A third, more common method, is not in this text. It is only hinted at in Example 5 on p. 242 and in the *Technical Notes* on pg. 245. P(A or B) = P(A) + P(B) − P(A and B).
      Those that have already learned this formula elsewhere are permitted to use it here:
      $1/6 + 5/6 - (1/6 \times 5/6) = 31/36$. (A and B are independent.)
      (Be aware that this formula only holds for combining two outcomes, while the de Méré method may be used for a chance when there are more than two outcomes being combined (P(A or B or C), for example).

    The solution shown below is for the de Méré method.

    Do not add the chances of events A and B: they are not mutually exclusive. A four could happen on the first roll with a diffrent result on the second roll.

    Instead find the chance of the opposite event to "A or B", and subtract that probability from 1 (or 100%). See the second box on page 223 and the box on page 250.

    State the opposite result: a non-four on the first roll and the same result on the second roll.

    Events A and B are indepndent: it does not matter whether or not a four happened on the first roll, the chances of not getting a different number on the second roll will be 1/6.

    The chance of the opposite is P(not A) × P(not B).

    That is $5/6 \times 1/6$ using the multiplication rule for independent events (on page 232).

    The chance of the opposite is 5/36; so the chance of A or B is $1 - 5/36 = 31/36$, answer (F).

21. (B) 3%

This is a binomial probability, with the clue the word "exactly."

However, most calculators will not return the value of 350 factorial (350!).

The normal approximation to the probability histogram for the binomial should be used, as the number of tosses is large enough.

Set up the box model: one 0 and one 1. The average and the SD of this box are both 1/2.

The number of heads is modeled by the sum of the draws from this 0–1 box.

Find the EV for the sum of the draws and the SE for the sum of draws.

They are: $350 \times 0.5 = 175$, and $\sqrt{350} \times 0.5 = 9.354$.

The box of the probability histogram for 182 heads runs from 181.5 to 182.5 heads.

(See page 317, example 1(a).)

The standard units for the endpoints are:

$(181.5 - 175)/9.354 \approx 0.70$ and $(182.5 - 175)/9.354 \approx 0.80$.

The area under the normal curve between 0.70 and 0.80 = $57.63\%/2 - 51.61\%/2 = 3.01\%$.

22. Answer: (D).

For the difference between the number of heads observed and the expected number of heads, see figure 2 on page 276 and the last comment by *Assistant* on pages 276 to 277.

That difference tends to get larger as the number of tosses increases.

For the difference between the percentage of heads observed and 50%, see figure 1 on page 275 and the last comment by *Assistant* on pages 276 to 277.

That difference tends to get smaller as the number of tosses increases.

This agrees with (D).

23. Put 1's on the tickets that count for you, 0's on the others.

(Box on page 301 in the text.)

Answer: (B)

24. (D) 3.44%

To get the proper counting box needed to model the percentage, put 1 on the even-numbered cards and put 0 on all the other cards.

The give or take for the sample percentage is just the SE for the sample percentage, which has the formula: $\dfrac{\text{SD of the box}}{\sqrt{\text{no. of draws}}} \times 100\%$.

(See the technical note on page 362 of the text.)

Use the shortcut to find the SD of the box:

The fraction of tickets with 1 is 20/52, and the fraction of tickets with 0 is 32/32.

The formula gives: $(1-0)\sqrt{\frac{32}{52} \times \frac{20}{52}} = \sqrt{0.38462 \times 0.61538} = \sqrt{0.23669} = 0.4865$.

(See the formula on page 298.)

Then the answer is: $\frac{0.4865}{\sqrt{200}} \times 100\% = 3.44\%$, answer (D).

25. (D) 82.30%

    The average of the draws is the same as the sample average.

    The expected value for the average of the draws is equal to the average of the box, here 120. (See the box on page 410.)

    The standard error for the average is (using the technical note on page 415) the SD of the box, divided by the square root of the number of draws.

    Here it gives: $20/\sqrt{81} = 20/9 \approx 2.2222$.

    A sample of size 81 should be large enough for the normal approximation to apply.

    The standard units for a given sample average is found with the formula:

$$z = \frac{\text{given sample average} - \text{EV for sample average}}{\text{SE for sample average}}.$$

    Find the standard units for 117 and 123: $(117 - 120)/2.2222 = -1.35$ and $(123 - 120)/2.2222 = 1.35$.

    The area under the normal curve from -1.35 to $+1.35$ is just the table entry: 82.30%, answer (D).

26. Only (E) is correct.

    (A) The SE average is $0.717/\sqrt{900} = 0.239$, not 0.717.

    (B) The 95% confidnce interval is plus or minus *two* SDS: 0.5822 to 0.6778.

    (C) The number of cats is not normally distributed, and no one can have a fractional number of cats anyway.

    (D) The number of cats does not follow the normal curve. There is a long right tail.

    (E) True. That is the central limit theorem (see p-age 325.)

27. **WARNING** This problem is one of the rare algebra problems in the course.

    The chance error for the percent is (SD of box)/$\sqrt{\text{no. of draws}}$ × 100%.

    The SD of a 0-1 box cannot exceed 0.50, so use the worst case in the calculation.

    $0.25\% = 0.50/\sqrt{n} \times 100\%$.

    $50\%/\sqrt{n} = 0.25\%$

    $50\% = \sqrt{n}$ times 0.25%

    $\sqrt{n} = 50\%/0.25\% = 200$

    Now square both sides. $n = 200^2 = 40,000$. Answer: (D) 40,000

28. Answer: (C) 15

    See the second box on page 412, the square-root law.

29. Since there is no trend or pattern to the data, a box model applies.

    The weight of a package is the sum of the 4 draws. The give or take applies to the sum of the draws.

    (See both boxes on page 291 and the top box on page 292.)

    SE for the sum $= \sqrt{n} \times$ (the SD of the box) $= 2 \times 0.05$ ounces $= 0.1$ ounces. Answer: (D)

30. Since there is no trend or pattern to the data, a box model applies.

The total time of execution is the sum of the 400 draws. The give or take applies to the sum of the draws.

(See both boxes on page 291 and the top box on page 292.)

SE for the sum $= \sqrt{n} \times$ (the SD of the box) $= 20 \times 1.23$ seconds $= 24.6$ seconds. Answer: (D)

31. First state the null and alternative hypotheses.

Null: The average of the box equals 20.

Alternative: The average of the box is more than 20.

Now, assume that the null is true, and decide if a result of the observed average or more extreme should be attributed to chance.

Because the draws are made at random from a box, the square root law applies. (Second box on page 291, applied to averages.) And because the number of draws is reasonably large, the normal curve may be used to get chances for the average of the draws. (See the box on page 412.)

Since the null hypothesis is assumed to be true, the EV for the sample average (average of the draws) is 20.

Now find the SE for the average of the draws.

First estimate the SD of the box by the SD of the sample, which is 10. (Box on page 416)

Then use the formula for the SE for the average of the draws found on the top of page 415: $10/\sqrt{100} = 20/10 = 1$.

The standard units come out to be:

$$\frac{\text{observed average} - \text{EV for average of draws}}{\text{SE for average of draws}} = \frac{22.7 - 20}{1} = 2.7.$$

P will be the area of the right tail: $(100\% - 99.31\%)/2 = 0.69\%/2 = 0.345\%$.

Since P is less than 5%, conclude that chance error is not a reasonable explanation and the average of the box is more than 20. Answer: (A)

32. (A) 1%, unfair

There are 9 degees of freedom: one less than the number of categories.

The expected values are all $1000 \times (1/10)$, or 100.

For each category, subtract the expected value from the observed, square the result, and then divide by 100, the expected.

Here is the first calculation: $\frac{(117-100)^2}{100} = 289/100 = 2.89$.

Add the ten results.

This is χ^2: $2.89 + 0.04 + 0.25 + 4.00 + 0.64 + 1.96 + 4.41 + 5.76 + 0.04 + 1.69 = 21.68$.

Or, since they are all over 100, add the six squares of the (observed − expected) and then divide by 100: $\frac{289+4+25+400+64+196+441+576+4+169}{100} = \frac{2168}{100} = 21.68$.

From the χ^2-table, the value of P is estimated at 1%.

Since $P = 1\%$, we do not attribute the difference to chance error. Instead, we reject the null hypothesis and conclude that the die is unfair.

Answer: (A) 1%, unfair