Class Worksheet February 5 to 12

Math 125 Kovitz 2025

The Normal Approximation for Data

The Normal Curve.



A normal curve with a horizontal scale in standard units is symmetric about 0, has under it a total area of 100%, is always above the horizontal axis, and gets and remains very close to the horizontal axis between 3 and 4 and (in the negative direction) between -3 and -4.

A value is converted to standard units by seeing how many SDs it is above or below the average.

The formula is: standard units $=\frac{\text{observation} - \text{average}}{\text{SD}}$.

Finding Areas under the Normal Curve.

An area from minus a value to plus the same value is read off from the Normal Table on page A-104; other areas are found by making a sketch and expressing the desired area in terms of areas that may be found by using the Table.

The Normal Approximation for Data.

If a histogram follows the normal curve, approximate areas may be found by converting the endpoints to standard units and finding the appropriate areas under the normal curve by using the Table.

Percentiles

The *n*th percentile is a value in the distribution for which n% of the values in the distribution were equal to or smaller than that value. For example, the 23rd percentile of a distribution of family incomes is a family income (say \$17,349). That means that 23% of all family incomes were equal to or smaller than \$17,349.

A percentile is not a percent, but a number which has the given percentage of the area in the distribution below it. That number puts you at the nth percentile of the number distribution. (The percentile rank is a percent, the percent associated with that number.)

Percentiles are often used for distributions with a long tail.

The interquartile range is defined as the value

75th percentile -25th percentile.

It is sometimes used as a measure of spread, when the SD would pay too much attention to a small percentage of cases in the tail of a distribution.

Percentiles and the Normal Curve

When calculating percentiles for a histogram that follows the normal curve, remember that the normal table cannot be used directly because it gives areas between -z and z, while a percentile involves the area to the left of z.

Change of Scale

Add (change average but not SD), multiply (change average and SD). In both cases the standard units stay the same.

Problems to think about

On a certain exam, the average of the scores was 40, and the SD was 8.

- (a) Convert each of the following scores to standard units: 44, 30, 60.
- (b) Find the scores which in standard units are: 0, +1.8, -2.2.

Here is a list of numbers:

30, 18, 22, 14, 6.

- (a) Convert each entry on the list to standard units (using, of course, the average and SD of the list).
- (b) Find the average and SD of the converted list.

CLASS WORKSHEET–FEBRUARY 5 to 12

Find the area under the normal curve—

- (a) to the right of 1.45 (b) to the left of -0.30
- (c) to left of 0.90 (d) to the right of -0.95
- (e) between -1.85 and +1.85 (f) between 0.20 and 1.10
- (g) between -0.40 and 0.80 (h) between -1.40 and -0.45
- (i) outside -1.7 to 1.7

Fill in the blanks:

- (a) The area between \pm _____ under the normal curve equals 77%.
- (b) The area between \pm _____ under the normal curve equals 95%.

The normal curve is sketched below; solve for z.

(a)
$$= 92\%$$
 (b) $= 96\%$

For the men in a certain health survey, the average height was about 68.2 inches; the SD was about 2.8 inches. Using the normal curve, estimate the percentage of men with heights—

- (a) below 70 inches
- (b) between 64 inches and 70 inches
- (c) above 72 inches

In a certain college, entering freshmen averaged 560 on the math SAT; the SD was about 90. The histogram of the math SAT scores followed the normal curve reasonably well.

- (a) About what percentage of entering freshmen scored below 695?
- (b) One student was 0.85 SDs above average on the math SAT. About what percentage of entering freshmen had lower scores than she did?

Among freshman entering a certain college, the math SAT scores averaged 560, the SD was 90, and the scores followed the normal curve.

- (a) Estimate the 90th percentile of the score distribution.
- (b) Estimate the 20th percentile of the score distribution.
- (c) One freshman scored 700 on the math SAT. What was her percentile rank (among all the freshmen)?

A distribution follows the normal curve. Estimate—in standard units its interquartile range.