Class Worksheet

March 7 and 10

Math 125 *Kovitz* 2025

More about Chance

Listing the Ways

When each of a number of ways something can turn out is equally likely as any other, it is often useful to count up the total number of ways. Using this total, the number of ways that constitute a given thing is then divided by the total number of ways to find the probability of that thing.

The Addition Rule

Two things are *mutually exclusive* when the occurrence of one prevents the occurrence of the other: one excludes the other.

To find the chance that at least one of two things will happen, check to see if they are mutually exclusive. If they are, add the chances.

In this case: " . . . the chance that A *or* B will happen . . . "

(or restated:) " . . . the chance that either A *or* B or both will happen . . . "

(Conjunction: **or**)

If you want to find the chance that at least one event occurs, and the events are not mutually exclusive, do not add the chances: the sum will be too big.

Blindly adding the chances can give the wrong answer, by double-counting the chance that both things happen. With mutually exclusive events, there is no double-counting.

Finding the Chance that at Least One of Several Things Will Happen

Special Case: When the things are mutually exclusive.

The chance that at least one of those things will happen reduces to the chance that exactly one of them will happen—the things being mutually exclusive means that there is no chance that more than one of them will happen. This chance can be calculated by the addition rule.

General Case: When the things are not mutually exclusive.

The opposite thing is that none of those things will happen.

This, the opposite thing, means that each of the original things did *not* happen. Its chance can now be calculated by repeated application of the multiplication rule.

The original chance that we desired—namely the chance that at least one of several things will happen—can now be found by subtracting the chance of its opposite from 100%.

The Real World

In many cases in the real world, each of several results of a chance process may be assumed to be *approximately* equally likely. If the discrepancy is small, any calculations based on that assumption will be just about correct.

Problems to think about

What is the chance of throwing a total of 6 spots with two dice?

Among the students in a class of thirty students eleven understand French and seven understand Vietnamese. True or false: Eighteen students understand French or Vietnamese. Explain briefly.

A fair coin is tossed twice. In each part tell whether those two things listed in that part are mutually exclusive or not.

(a) The two things are:

Getting a head on the first toss
and
getting a head on the second toss.

(b) The two things are:

Getting a total of two heads on the two tosses
and
getting a total of two tails on the two tosses.

A fair coin is tossed twice. To calculate the chance of getting a head on the first toss or the second toss or both, I note that the chance of getting a head on the first toss is 50% and the chance of getting a head on the second toss is 50%. I then propose to add the two chances to get 100% as the chance of getting a head on the first toss or the second toss or both. Am I reasoning correctly?

A fair coin is tossed twice. To calculate the chance of getting two heads or two tails or both on the two tosses, I note that the chance of getting two heads on the two tosses is 25% and the chance of getting two tails on the two tosses is 25%. I then propose to add the two chances to get 50% as the chance of getting either two heads or two tails or both on the two tosses. Am I reasoning correctly?

A box contains 10 tickets numbered 1 through 10. Seven draws will be made at random with replacement from this box. True or false: There are 7 chances in 10 of getting $\boxed{9}$ at least once. Explain briefly.

Same as previous problem except the drawing is without replacement.

The unconditional probability of event A is 2/5. The unconditional probability of event B is 1/4.

(a) Assuming that A and B are independent, find the chance that both happen.

(b) Assuming that A and B are mutually exclusive, find the chance that neither happens.

A game of chance starts with a six-shooter containing one live bullet and five blanks. A contestant spins the barrel, randomly selecting a chamber, and then fires. If the bullet proves to be live he wins \$10.

He then gets to fire again for a chance to win another \$10.

True or false: The chance that he wins \$10 at least once in the two games is $1/6 + 1/6$ if:

(a) he discharges the next chamber without spinning in between.

(b) he reloads the same type of bullet that was originally there before the first shot (if he won: another live one), spins, and fires for the second time.

Point out any errors in these two proposed solutions to the above problem:

- In (a), the chance is $1/6 + 1/5 = 11/30$, because they are mutually exclusive.
- In (b), it's $1/6 + 1/6$, because each chance is $1/6$ and they're independent.

A deck with five cards labeled A through E is shufffled. True or false, and explain briefly.

(a) The chance that the top card is the D equals 1/5.

(b) The unconditional chance that the second card is the C equals 1/5.

(c) The chance that the top card is the D or the second card is the C equals 2/5.

(d) The chance that the top card is the D or the second card is the D equals 2/5.

A box has five balls: four blue and one orange. Three draws are made at random from the box without replacement.

(a) Find the chance of getting the orange ball on any given draw.

(b) True or false: The chance of getting the orange ball at least once in the three draws is $1/5 + 1/5 + 1/5$. If false, calculate the correct chance.

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(a) Find the chance of getting the orange ball on any given draw.

(b) True or false: The chance of getting the orange ball at least once in the three draws is $1/5 + 1/5 + 1/5$. If false, calculate the correct chance.

Suppose that of all students at UMass/Boston, 39% live in Boston, 9% live in Cambridge, and 6% live in Quincy. Find the chance that a student selected at random lives in at least one of those three cities.

Suppose that of all students at UMass/Boston, 56% are female and 59% are over 21, and sex and age are independent. Find the chance that a student selected at random falls into at least one of the 2 categories: female or over 21.

A die is rolled ten times. What is the chance of never getting a six?

A die is rolled five times. What is the chance of getting at least one number exactly divisible by three?

Five digits, each from 0 to 9, are chosen randomly as if five tickets were drawn with replacement from a box with the tickets numbered from 0 to 9, inclusive. What is the chance that at least one of the digits chosen is more than 6 (that is 7, 8, or 9)?

A certain panhandler calculates that about 1% of the people he approaches will give him one dollar or more. Find the chance that at least one of 150 individuals, selected randomly and independently, will give him one dollar or more.

In the Massachusetts State Lottery's Daily Number Game®, is it reasonable to assume that, when the first digit is drawn, the numbers 0 to 9 have equal chance? Do you really think that the chances are precisely equal or only approximately equal? What about any calculations based on the assumption of equality?

Formulas

The Chance that both of two things will happen:

General case:

$$\text{P(A and B)} = \text{P(A)P(B|A)}$$

Special case when A and B are independent (meaning that P(B|A) = P(B)):

$$\text{P(A and B)} = \text{P(A)P(B)}$$

The Chance that at least one of two things will happen:

General case (optional):

$$\text{P(A or B)} = \text{P(A)} + \text{P(B)} - \text{P(A and B)}$$

Special case when A and B are mutually exclusive (meaning that P(A and B) = 0):

$$\text{P(A or B)} = \text{P(A)} + \text{P(B)}$$