Class Worksheet March 26 Math 125 Kovitz 2025

Chance Variability

The Law of Averages

Chance Processes (those that produce a number as a result of chance)

In many cases the process being studied can be made analagous to the process of drawing numbers from a box. This analogy is called a box model.

When a process analogous to drawing from a box is repeated a large number of times, the following formula applies to the number of times that a certain result will occur among the trials

number of results observed = expected number + chance error.

As the number of trials increases, the absolute value of the above-described chance error tends to get bigger and the chance error as a percentage of the number of tosses tends to get smaller.

In a situation involving gain or loss as a result of a random process, one may make a box model.

The tickets in the box show the various amounts that can be won or lost on a single play.

The chance of drawing any particular number from the box must equal the chance of winning that amount on a single play. ("Winning" a negative amount is the mathematical equivalent of what most people call losing.)

The number of draws equals the number of plays.

The *sum of draws* from a box is shorthand for the processs of drawing tickets at random from a box and then adding up the numbers on the tickets. The formula that applies is

observed sum of the draws = expected sum + chance error.

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Problems to think about

A machine has been designed to toss a coin automatically and keep track of the number of heads. After 10,000 tosses, it has 5097 heads. Express the chance error both in absolute terms and as a percentage of the number of tosses.

A coin will be tossed, and you win a dollar if the number of heads is exactly equal to the number of tails. Which is better: 10 tosses or 50 tosses? Or are they the same? Explain.

A coin will be tossed, and you win a dollar if the percentage of heads is between 45% and 55%. Which is better: 100 tosses or 900 tosses? Explain.

A coin will be tossed, and you win a dollar if there are more than 55% heads. Which is better: 100 tosses or 900? Explain.

One ticket will be drawn at random from the box $\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} \begin{vmatrix} 4 & 4 \\ 4 & 4 \end{vmatrix}$.

What is the chance that it will be 0? that it will be 2 or less? 3 or more?

A quiz has 10 true-false questions: a correct answer is worth 10 points, but 10 points are taken off for each incorrect answer. A student answers all the questions at random, as if by tossing a coin. The student's total score is like the sum of ______ draws made at random with replacement from the box ______. Fill in the blanks; explain.

A person wagers two dollars that the first digit of the next number drawn in the Daily Number Game^(R) will be eight. If the first digit turns out to be eight, he will win ten dollars. If the first digit turns out to be one of the nine other possibilities, he will lose two dollars. He repeats this wager for thirty-one days. His total profit or loss for the thirty-one days is like the sum of ______ draws made at random with replacement from the box ______. Fill in the blanks; explain. What do you think of his prospects?

(The first digit in the Daily Number Game (\mathbb{R}) is an integer from 0 to 9, chosen independently and at random.)