Class Worksheet

March 28 and 31
Math 125 *Kovitz* 2025

The Expected Value and Standard Error

The Expected Value

The expected value for the sum of draws made at random with replacement from a box equals

$$(\text{number of draws}) \times (\text{average of box}).$$

The Standard Error

A sum is likely to be around its expected value, but to be off by a chance error similar in size to the standard error.

When drawing at random with replacement from a box of numbered tickets, the standard error for the sum of the draws is

$$\sqrt{\text{number of draws}} \times (\text{SD of box}).$$

Using the Normal Curve to Find the Chance that the Sum of a Large Number of Draws (made at random with replacement from a box) will be in a Given Range

First find the expected value and standard error for the sum of draws.

Next convert the endpoints of the given range to standard units using the formula

$$\text{standard units} = \frac{\text{given value} - \text{expected value}}{\text{standard error}}.$$

The area under the normal curve between the standard units for the endpoints of the given range will be an approximation for the desired chance.

A Shortcut to Find the SD of a Box with Only Two Kinds of Tickets

When the tickets in the box show only two different numbers, the SD of the box equals

$$\begin{pmatrix}\text{bigger}\\ \text{number}\end{pmatrix} - \begin{pmatrix}\text{smaller}\\ \text{number}\end{pmatrix} \times \sqrt{\begin{matrix}\text{fraction with}\\ \text{bigger number}\end{matrix} \times \begin{matrix}\text{fraction with}\\ \text{smaller number}\end{matrix}}$$

Classifying and Counting

If you have to classify and count the draws, put 0's and 1's on the tickets. Mark 1 on the tickets that count for you, 0 on the others.

Then find the average of the box and the expected value, and the SD (using the shortcut) and the standard error. Now convert any given values of the count to standard units and use the normal curve to approximate any chance being sought.

Problems to think about

Find the expected value for the sum of 100 draws at random with replacement from the box—

(a) $\boxed{0}\ \boxed{3}\ \boxed{4}\ \boxed{9}$ (b) $\boxed{-3}\ \boxed{-1}\ \boxed{0}\ \boxed{3}$

(c) $\boxed{-5}\ \boxed{-2}\ \boxed{7}$ (d) $\boxed{0}\ \boxed{1}\ \boxed{3}$

One hundred draws are made at random with replacement from the box $\boxed{4}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{10}$.

(a) Find the expected value and standard error for the sum.

(b) Suppose you had to guess what the sum was going to be. What would you guess? Would you expect to be off by around 2, 4, 20, or 40?

You gamble 100 times on the toss of a coin. If it lands heads, you win \$5. If it lands tails, you lose \$1. Your net gain will be around _____, give or take _____ or so. Fill in the blanks, using the options

$-\$200$ $-\$50$ $-\$30$ $-\$10$ $-\$5$ $\$0$ $\$5$ $\$10$ $\$30$ $\$50$ $\$200$

Four hundred draws will be made at random with replacement from the box $\boxed{1}\ \boxed{11}\ \boxed{17}\ \boxed{19}$.

(a) The smallest the sum can be is _____, the largest is _____.

(b) The sum of the draws will be around _____, give or take _____ or so.

(c) The chance that the sum will be bigger than 5000 is about _____%.

(d) How likely is the sum to be in the range from 4520 to 4960?

Find the SD of the box—

(a) $\boxed{2}\ \boxed{2}\ \boxed{2}\ \boxed{-1}\ \boxed{-1}$

(b) $\boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{9}$

A biased coin lands heads with chance 0.6. That coin is tossed 10,000 times.

(a) The number of heads is like the sum of 10,000 draws made at random with replacement from one of the following boxes. Which one and why?

(i) $\boxed{\text{head}}\ \boxed{\text{head}}\ \boxed{\text{head}}\ \boxed{\text{tail}}\ \boxed{\text{tail}}$ (ii) $\boxed{\text{head}}\ \boxed{\text{tail}}$

(iii) $\boxed{0}\ \boxed{1}$ (iv) $\boxed{0}\ \boxed{0}\ \boxed{1}\ \boxed{1}\ \boxed{1}$

(b) The number of heads will be about _____, give or take _____ or so.

Four hundred draws will be made at random with replacement from the box $\boxed{1}\ \boxed{11}\ \boxed{17}\ \boxed{19}$. What is the chance of getting between 74 and 126 tickets marked 17?

A large group of people get together. Each one rolls a die 720 times, and counts the number of ⚀ 's. About what percentage of these people should get counts in the range 105 to 135?