Class Worksheet

April 25 and 28
Math 125 *Kovitz* 2025

Chance Models

Measurement Error **(Chapter 6 review)**

Chance Error

No matter how carefully it was made, a measurement could have come out a bit differently than it did. If it is repeated, it will come out a bit differently. By how much? Before relying on any measurement, this question must be faced. The best way to answer it is by replicating the measurement.

The SD of a series of repeated measurements is an estimate of the likely size of the chance error in a single measurement.

Outliers should not be discarded; they are actual results of the real process.

Bias

Bias affects all measurements the same way, pushing them in the same direction. Chance errors change from measurement to measurement, sometimes up and sometimes down.

Individual measurement = exact value + bias + chance error.

A Model for Measurement Error **(Chapter 24)**

Estimating the accuracy of an average.

The chances are in the measuring procedure, not in the thing being measured.

If the data show a trend or pattern over time, a box model does not apply.

The square root law only applies to draws from a box.

The Gauss model applies to repeated measurements on some quantity. According to the model, each time a measurement is made, a ticket is drawn at random with replacement from the error box. The number on the ticket is the chance error. It is added to the exact value to give the actual measurement. The average of the error box is equal to 0.

When the Gauss model applies, the SD of a series of repeated measurements can be used to estimate the SD of the error box. The estimate is good when there are enough measurements.

If bias is present, the SE for the average says how far the average of the measurements is likely to be from the exact value + the bias, instead of from the exact value.

Problems to think about

A yardstick is carefully measured 100 times. The sequence of measurements averages out to 91.362 cm, and the SD is 640 microns. (A *micron* is the millionth part of a meter.)

(a) Is a single reading off by around 64 microns, or 640 microns?

(b) Is the average of all 100 readings off by around 64 microns, or 640 microns?

(c) Find a 95%-confidence interval for the exact length of the yardstick.

The 95%-confidence interval for the exact weight of a certain check weight is the range from 186.8 to 188.4 micrograms below 100 grams, based on 100 weighings. Say whether each of the following statements is true or false, and explain why.

(a) About 95% of the measurements are in this range.

(b) There is about a 95% chance that the next measurement will be in this range.

(c) About 95% of the time that the technician takes 100 measurements and goes 2 SEs either way from the average, he succeeds in covering the exact weight.

(d) If the technician took another 100 measurements on the check weight, there is about a 95% chance that the new average would fall in the interval from 186.8 to 188.4 micrograms below 100 grams.

In Rapid City, South Dakota, in an average year about one day in three is rainy, about one day in three is partly cloudy, and about one day in three is sunny. Someone proposes the following chance model for the sequence of rainy, partly cloudy, and sunny days: draw with replacement from a box containing one card marked "rainy," one card marked "partly cloudy," and one card marked "sunny." Is this a good model?

A 20-gram check weight is being weighed. Assume the Gauss model with no bias. If the exact weight is 402 micrograms above 20 grams, and the number drawn from the error box is 4 micrograms, what would the measurement be?

The Bureau is about to weigh a one-kilogram check weight 400 times, and take the average of the measurements. They are willing to assume the Gauss model, with no bias, and on the basis of past experience they estimate the SD of the error box to be 80 micrograms.

(a) The average of all 400 measurements is likely to be off the exact weight by _____ or so.

(b) The SD of all 400 measurements is likely to be around _____.

(c) Estimate the probability that the average of all 400 measurements will be within 6 micrograms of the exact weight.

Sixty-four measurements are made on the speed of light. These average out to 300,006.8 and the SD is 10, the units being kilometers per second. Fill in the blanks in part (a), then say whether each of (b–f) is true or false; explain your answers briefly. (You may assume the Gauss model, with no bias.)

(a) The speed of light is estimated as _____ ; this estimate is likely to be off by _____ or so.

(b) Each measurement is off 300,006.8 by 10 or so.

(c) The average of all 64 measurements is off 300,006.8 by 1.25 or so.

(d) If a 65th measurement were made, it would by off the exact value for the speed of light by 1.25 or so.

(e) A 95%-confidence interval for the speed of light is 300,006.8 ± 2.5.

(f) A 95%-confidence interval for the average of the 64 measurements is 300,006.8 ± 2.5.

(g) Because the Gauss model applies here, individual measurements will follow the normal curve.

(h) It's valid here to base a confidence interval on the normal curve.

You may assume the Gauss model for all five parts of this question. Say whether each assertion is true or false, and give a reason.

(a) If all you have is one measurement, you can't estimate the likely size of the chance error in it—you'd have to take another measurement, and see how much it changes.

(b) If all you have is one hundred measurements, you can't estimate the likely size of the chance error in their average—you'd have to take another hundred measurements, and see how much the average changes.

(c) The SD of the error box is exactly like the SE for the average of the draws from the error box for sample size one.

(d) The SD of the error box shows the likely amount that a single draw made at random from the error box will be off from the average of the error box.

(e) The SD of the error box shows the likely amount that a single measurement will be off from the exact value + the bias.