Class Worksheet

April 30 and May 2
Math 125 *Kovitz* 2025

Chance Models in Genetics **(Chapter 25)** (optional)

(This chapter is particularly useful for biologists and geneticists.)

How Mendel Discovered Genes

Did Mendel's Facts Fit His Model?

The Law of Regression

An Appreciation of the Model

Tests of Significance (Part VIII)

Tests of Significance **(Chapter 26)**

To make a test of significance, the null hypothesis has to be formulated as a statement about a box model. Usually, the alternative does too.

- The *null hypothesis* says that an observed difference just reflects chance variation.
- The *alternative hypothesis* says that the observed difference is real.

> The null hypothesis expresses the idea that an observed difference is due to chance. To make a test of signficance, the null hypothesis has to be set up as a box model for the data. The alternative hypothesis is another statement about the box; it says that the difference is real.

A test statistic is used to measure the difference between the data and what is expected on the null hypothesis.

z says how many SEs away an observed value is from its expected value, where the expected value is calculated using the null hypothesis. The formula is

$$z = \frac{\text{observed value} - \text{expected value}}{\text{standard error}}.$$

The observed significance level—often called the P-value—is the chance of getting a test statistic as extreme as or more extreme than the observed one. The chance is computed on the basis that the null hypothesis is right. The smaller the chance is, the stronger the evidence against the null.

A small value of P indicates that an explanation saying that this is chance variation is unreasonable. We cannot accept the model stated in the null.

A large value of P could very well be due to chance variation and we accept the null as a reasonable model.

The P-value of a test is the chance of getting a big test statistic—assuming the null hypothesis to be right. P is not the chance of the null hypothesis being right.

Making a Test of Significance

Based on some available data, the investigator has to—

- translate the null hypothesis into a box model for the data;
- define a test statisic to measure the difference between the data and what is expected on the null hypothesis;
- compute the observed significance level P.

The choice of test statistic depends on the model and the hypothesis being considered.

Many statisticians have a dividing line that indicates how small the observed significance level has to be before an investigator should reject the null hypothesis.

- If P is less than 5%, the result is called *statistically significant*.
- If P is less than 1%, the result is called *highly significant*.

Tests of Significance Using Zero-one Boxes **(Chapter 26, section 5)**

If a problem involves classifying and counting, the z-test can be used. The box will contain 0's and 1's.

Once the null hypothesis has been translated into a box model, it is easy to test using

$$z = \frac{\text{observed} - \text{expected}}{\text{SE}}$$

Problems to think about (Chapter 26)

A die is rolled 10,000 times. The total number of spots is 35,221, instead of the expected 35,000. Can this be explained as a chance variation, or is the die loaded?

True or false: The alternative hypothesis says that nothing is going on, besides chance variation.

True or false: In order to test a null hypothesis, this hypothesis has to be translated into a statement about a box model.

A test of significance makes sense in an argument about ________ .

data a model for data

Choose one option, and explain briefly.

A hundred draws are made at random with replacement from a box. The average of the draws is 87.2, and their SD is 20. Someone claims that the average of the box equals 80. Is this plausible? What if the average of the draws is 81.6, and their SD is 20?

According to one investigator's model, the data are like 1600 draws made at random from a large box. The null hypothesis says that the average of the box equals 180, the alternative says that the average of the box is more than 180. In fact, the data averaged out to 183.9, and the SD was 60. Compute z and P. What do you conclude?

True or false:

(a) The observed significance level depends on the data.

(b) A "highly significant" result cannot possibly be due to chance.

(c) If a difference is "highly significant," there is less than a 1% chance for the null hypothesis to be right.

(d) If P is 74%, the null hypothesis looks plausible.

(e) If P is 0.74 of 1%, the null hypothesis looks implausible.

(f) If $z = 2.6$, then the observed value is 2.6 SEs above what is expected on the null hypothesis.

An investigator draws 900 tickets at random with replacement from a box. What is the chance that the average of the draws will be more than 2 SEs above the average of the box?

To test for ESP, a subject is asked to guess the value of one of 5 randomly selected targets. Suppose that in 900 trials, he scores 217 correct guesses.

(a) Set up the null hypothesis as a box model.

(b) The SD of the box is ________ . Fill in the blank, using one of the options below, and explain briefly.

$$\sqrt{0.2 \times 0.8} \qquad\qquad \sqrt{0.241 \times 0.759}$$

(c) Make the z-test.

(d) What do you conclude?