Scientific Notation and Related Ideas

(Basic Algebra Review)
Math 130 *Kovitz*

Decide whether each of Problems 1 to 22 is true or false. Assume m and n are integers.

1. $a \times 10^{-n}$ will have n leading zeros after the decimal point in the case when $-n$ is negative.
2. $a \times 10^{m}$ has $m+1$ digits before the decimal point when m is non-negative.
3. The scientific notation of 1 is 1×10^0; the scientific notation for a real number that is greater than or equal to 1 will be $a \times 10^n$ with $n \geq 0$; and the scientific notation for a real number that is less than 1 will be $a \times 10^n$ with $n < 0$ (n negative).
4. A real number r with $1 \leq r < 10$ is represented in scientific noatation as $r \times 10^0$.
5. Some real numbers have two different representations in scientific notation, i.e. $a \times 10^n$ and $b \times 10^m$ with either $a \neq b$ or $n \neq m$.
6. $0 = 0.0 \times 10^0$ is the scientific notation of zero.
7. Multiplying 6×10^4 by 2×10^{11} yields 1.2×10^{16}.
8. Finding $\frac{1}{2}$ of 6.4×10^9 is correctly accomplished by observing that $.5 \times 10^{-1}$ times 6.4×10^9 equals 3.2×10^8.
9. To multiply a number $a \times 10^n$ that is in scientific notation by a simple fraction, it is permissible to simply multiply a by that fraction; and an example is that $\frac{1}{8}$ of 9.6×10^{-3} should equal 1.2×10^{-3}.
10. $2 \times 10^{-7} + (2 \times 10^{-7}) \times (5 \times 10^3) = (4 \times 10^{-7})(5 \times 10^3)$.
11. $2 \times 10^{-7} + (2 \times 10^{-7}) \times (5 \times 10^3) = (2 \times 10^{-7})(5.01 \times 10^3)$.
12. $\dfrac{6 \times 10^{-5}}{8 \times 10^{-3}} = 7.5 \times 10^{-2}$.
13. A proper solution for the reciprocal of $a \times 10^n$ is $\dfrac{1}{a \times 10^n} = \dfrac{10}{a} \times 10^{-n-1} = \dfrac{10}{a} \times 10^{-(n+1)}$.
14. The reciprocal of $\frac{4}{3} \times 10^{-2}$ in scientific notation is 0.75×10^2.
15. The reciprocal of $\frac{8}{7} \times 10^0$ in scientific notation is 8.75×10^{-1}.
16. $(7 \times 10^{-2})^2 = 4.9 \times 10^{-3}$.
17. The square root of 1.6×10^{-11} in scientific notation is 4×10^{-5}.
18. $\sqrt{10^n} = 10^{\sqrt{n}}$ for all integers n greater than or equal to 0.
19. In general, the square root of $a \times 10^{2n}$ with n an integer is $\sqrt{a} \times 10^n$, and the equare root of $a \times 10^{2n+1}$ with n an integer is $\sqrt{10a} \times 10^n$.
20. Adding 4.2×10^3 and 6.1×10^3 gives 1.03×10^4.
21. Adding 4.2×10^{-4} and 5.3×10^{-6} gives 4.253×10^{-4}.
22. $(1.801 \times 10^4) - 1 = 1.8009 \times 10^4$.

Problems 23 and 24 follow.

23. Using scientific notation whenever helpful, show that the square of $a \times 10^{-n}$ (aasuming $-n < 0$) must be less than $a \times 10^{-n}$ itself. For this example, assume that $a \times 10^{-n}$ is in scientific notation.

24. Using scientific notation when advisable, find $\sqrt{1 - r^2}$ when $r = \frac{7}{25}$. Do not use a calculator for this problem.

Answers follow.

Answers

1. False.
2. True.
3. True.
4. True.
5. False.
6. False.
7. True.
8. False.
9. True.
10. False
11. True.
12. False.
13. True.
14. Fa;se.
15. True.
16. True.
17. False.
18. False.
19. True.
20. True.
21. True.
22. True.
23. $a < 10$

    $a^2 < 10a$

    $(a \times 10^{-n})^2 < 10a \cdot 10^{-2n} < a \cdot 10^{-2n+1} < a \cdot 10^{-n}$.

    QED *(quod erat demonstrandum)*

24. $r = \frac{14}{5} \times 10^{-1}$

$$r^2 = \frac{196}{25} \times 10^{-2}$$

$$r^2 = \frac{196}{2500}$$

$$1 - r^2 = \frac{2304}{2500}$$

$$\sqrt{1 - r^2} = \frac{\sqrt{2304}}{\sqrt{2500}} = \frac{\sqrt{4}\sqrt{576}}{50} = \frac{\sqrt{576}}{25} = \frac{\sqrt{4}\sqrt{144}}{25} = \frac{2(12)}{25} = \frac{24}{25}.$$