Scientific Notation and Related Ideas

(Basic Algebra Review) Math 130 Kovitz

Decide whether each of Problems 1 to 22 is true or false. Assume m and n are integers.

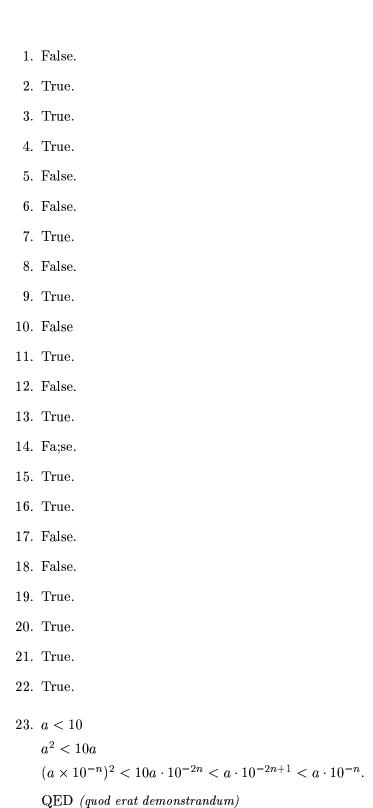
- 1. $a \times 10^{-n}$ will have n leading zeros after the decimal point in the case when -n is negative.
- 2. $a \times 10^m$ has m+1 digits before the decimal point when m is non-negative.
- 3. The scientific notation of 1 is 1×10^0 ; the scientific notation for a real number that is greater than or equal to 1 will be $a \times 10^n$ with $n \ge 0$; and the scientific notation for a real number that is less than 1 will be $a \times 10^n$ with n < 0 (n negative).
- 4. A real number r with $1 \le r < 10$ is represented in scientific notation as $r \times 10^0$.
- 5. Some real numbers have two different representations in scientific notation, i.e. $a \times 10^n$ and $b \times 10^m$ with either $a \neq b$ or $n \neq m$.
- 6. $0 = 0.0 \times 10^0$ is the scientific notation of zero.
- 7. Multiplying 6×10^4 by 2×10^{11} yields 1.2×10^{16} .
- 8. Finding $\frac{1}{2}$ of 6.4×10^9 is correctly accomplished by observing that $.5 \times 10^{-1}$ times 6.4×10^9 equals 3.2×10^8 .
- 9. To multiply a number $a \times 10^n$ that is in scientific notation by a simple fraction, it is permissible to simply multiply a by that fraction; and an example is that $\frac{1}{8}$ of 9.6×10^{-3} should equal 1.2×10^{-3} .
- 10. $2 \times 10^{-7} + (2 \times 10^{-7}) \times (5 \times 10^{3}) = (4 \times 10^{-7})(5 \times 10^{3}).$
- 11. $2 \times 10^{-7} + (2 \times 10^{-7}) \times (5 \times 10^3) = (2 \times 10^{-7})(5.01 \times 10^3)$.
- 12. $\frac{6 \times 10^{-5}}{8 \times 10^{-3}} = 7.5 \times 10^{-2}$.
- 13. A proper solution for the reciprocal of $a \times 10^n$ is $\frac{1}{a \times 10^n} = \frac{10}{a} \times 10^{-n-1} = \frac{10}{a} \times 10^{-(n+1)}$.
- 14. The reciprocal of $\frac{4}{3} \times 10^{-2}$ in scientific notation is 0.75×10^{2} .
- 15. The reciprocal of $\frac{8}{7} \times 10^0$ in scientific notation is 8.75×10^{-1} .
- 16. $(7 \times 10^{-2})^2 = 4.9 \times 10^{-3}$.
- 17. The square root of 1.6×10^{-11} in scientific notation is 4×10^{-5} .
- 18. $\sqrt{10^n} = 10^{\sqrt{n}}$ for all integers n greater than or equal to 0.
- 19. In general, the square root of $a \times 10^{2n}$ with n an integer is $\sqrt{a} \times 10^n$, and the equare root of $a \times 10^{2n+1}$ with n an integer is $\sqrt{10a} \times 10^n$.
- 20. Adding 4.2×10^3 and 6.1×10^3 gives 1.03×10^4 .
- 21. Adding 4.2×10^{-4} and 5.3×10^{-6} gives 4.253×10^{-4} .
- 22. $(1.801 \times 10^4) 1 = 1.8009 \times 10^4$.

Problems 23 and 24 follow.

- 23. Using scientific notation whenever helpful, show that the square of $a \times 10^{-n}$ (assuming -n < 0) must be less than $a \times 10^{-n}$ itself. For this example, assume that $a \times 10^{-n}$ is in scientific notation.
- 24. Using scientific notation when advisable, find $\sqrt{1-r^2}$ when $r=\frac{7}{25}$. Do not use a calculator for this problem.

Answers follow.

Answers



24.
$$r = \frac{14}{5} \times 10^{-1}$$

$$r^2 = \frac{196}{25} \times 10^{-2}$$

$$r^2 = \frac{196}{2500}$$

$$1 - r^2 = \frac{2304}{2500}$$

$$\sqrt{1-r^2} = \frac{\sqrt{2304}}{\sqrt{2500}} = \frac{\sqrt{4}\sqrt{576}}{50} = \frac{\sqrt{576}}{25} = \frac{\sqrt{4}\sqrt{144}}{25} = \frac{2(12)}{25} = \frac{24}{25}.$$