Exponent Practice

(for Exponential Intake Test)
Math 130 *Kovitz*

1. Find $1.5^{1.5}$

   Give answer in radical form with a rational denominator.

2. Find $2.25^{2.25}$

   Give answer in radical form with a rational denominator.

3. Find $(1/4)^{1/4}$

   Give answer in radical form with a rational denominator.

4. Evaluate and give exact answers with a rationalized denominator. The ansers will have a radical in them.

   (a) $4^{1.25} - 4^{-.25}$
   (b) $4^{1.25} + 4^{-.25}$

5. True or false?

   (a) $16^{-7/8} = -16^{7/8}$
   (b) $16^{-7/8} = (-16)^{7/8}$
   (c) $16^{-7/8} = \dfrac{1}{16^{8/7}}$
   (d) $16^{-7/8} = \dfrac{1}{16}\sqrt{2}$

6. True or false?

   (a) The expression $\sqrt{25}$ has two values.
   (b) The square root of 49 has two values.

7. True or false?

   (a) The square root of 100 is 10, because 100 is positive.
   (b) The convention is that $\sqrt{16} = \pm 4$, because $(+4)^2 = 16$ and $(-4)^2 = 16$.

8. Solve.

   (a) $x^4 = 4$.
   (b) $x^3 = 3\sqrt{3}$.

9. Find $\sqrt[4]{|x|^{3/2}}$.

10. Find $\sqrt[4]{4096^{3/2}}$. Do not use a calculator, but use the fact that $4096 = 64^2$. Get the exact answer in radical form.

11. Find $\sqrt[4]{2^{39} \div 2^{11}}$. What are the sign(s) of the answer?

12. True or False? A) $\sqrt{1.23^2 - 0.23^2} = 1$; B) $\sqrt{1.23^2 \div 0.23^2} = 1$; C) $\sqrt{5^{1.23} - 5^{0.23}} = \sqrt{5}$; D) $\sqrt{4^{1.23} \div 4^{0.23}} = 2$; E) $\sqrt{(5^{1.23})^{0.23}} = 5^{\left(\frac{1.23 \times 0.23}{2}\right)}$.

13. Simplify to an exact fraction, not a decimal approximation.

$$\sqrt{\frac{2/3}{3/5} + \frac{5/12}{2/5} - 2}$$

14. Simplify, if possible: $\sqrt{25x^2 - 1}$.

15. Simplify: $\sqrt{9x^2 - 42x + 49}$. Be careful when stating the answer.

    To verify the solution, let $x = 2$ and plug it into the original formula and get a numerical solution. Then plug it into the simplified version proposed and get a numerical solution. If these two numbers are the same, the problem was solved correctly.

    Are these two numbers different? What went wrong in the solution? If that is the case, fix the answer.

    Comment: This was a difficult problem.

16. Solve for x: $2^x + 2^{-x} = 2$.

17. True or false?

    If you cube a number and square the result, it will give the same result as when you square the number and cube the result.

18. The number 7 is being raised to the power p^q.

    Write, if possible, a simplified form of this expression: one that does not need parentheses.

19. (a) Evaluate $a^{(b^c)}$ when $a = 2\sqrt{2}$, $b = 4/9$, and $c = 1/2$.
    (b) Evaluate $(a^b)^c$ when $a = 2\sqrt{2}$, $b = 4/9$, and $c = 1/2$.
    (c) Are the results the same?

20. Divide

$$\frac{2.0 \times 10^{-7}}{4.0 \times 10^{-2}}$$

    and put the answer into scientific notation.

21. Find the square root of 1.6×10^{35}.

22. Consider the expression

$$\frac{\sqrt{1-x^2}}{\sqrt{1-x}}.$$

(a) The expression is well defined as a real number over what domain?

Find the value of that expression for $x = -3/4$ and for $x = 7/9$.

(b) Simplify the expression, assuming that x is in the domain found above.

Use that simplified form to evaluate the expression when $x = -3/4$ and when $x = 7/9$.

Was it easier with the simplified version? Do the results agree?

23. (a) Find the domain of the expressions $\sqrt{|x|}$ and $|\sqrt{x}|$. (The domain is the set of all possible values of x for which the expression is well defined and yields a real number.)

(b) True or false: For all x in the domain of $|\sqrt{x}|$, $|\sqrt{x}| = \sqrt{|x|}$.

24. True or false:

$$\sqrt{x^2} = \sqrt{|x^2|} = \sqrt{|x|^2}.$$

25. Using the result of the previous problem, simplify the expression

$$\sqrt{x^2},$$

so that there is no square or square root in the answer.

Does the sign of the answer match with the necessary result? Does it make sense? Does it work when x is positive; does it work when x is negative?

26. Here is a proposed solution to the equation $a^4 = \frac{64}{81}$.

a^4 is a positive real number between 0 and 1.

$a^4 = \dfrac{(4)(16)}{3^4}$.

So, $a = \dfrac{\sqrt[4]{4} \cdot 2}{3} = \frac{2}{3}\sqrt{2}$.

Is this correct: yes or no?

27. Suppose that $0 < a < 1$. Which is bigger: $\sqrt{a}$ or a? Demonstrate it.

Answers follow.

Answers

1. $\frac{3}{4}\sqrt{6}$.

2. $\frac{81}{32}\sqrt{6}$.

3. $\sqrt{2}/2$

4. (a) $3.5\sqrt{2}$.
   (b) $4.5\sqrt{2}$.

5. (a) False. (b) False. (c) False. (d) True.

6. (a) False. (b) True.

7. (a) False; it is ± 10. (b) False; it designates the positive square root, so it is only $+4$.

8. (a) $\pm\sqrt{2}$ (b) $\sqrt{3}$ (no plus or minus)

9. $|x|^{3/8}$, also could be written as $\sqrt[8]{|x|^3}$ or $\left(\sqrt[8]{|x|}\right)^3$. There is a positive answer only.

10. $16\sqrt{2}$

11. 128. The fourth root symbol just like the square root symbol, designates the positive root only.

12. A) False. B) False. C) False. D) True. E) True.

13. 4/15

14. It cannot be simplified, though $\sqrt{(5x-1)(5x+1)}$ and $\sqrt{|5x-1|}\sqrt{|5x+1|}$ are equivalent forms. They are probably not any simpler than the original.

15. $|3x-7|$.

    Both plug-ins of $x = 2$ will give 1. If you got -1 from your formula, it meant you forgot the absolute value.

16. $x = 0$.

17. True.

18. It cannot be done.

19. (a) 2.
    (b) $\sqrt[3]{2}$.
    (c) No. Compare problem 18.

20. 5.0×10^{-6}.

21. 4×10^{17}.

22. (a) Over $[-1, 1)$, which is cc $-1 \leq x < 1$.
    1/2 and 4/3.

    (b) $\sqrt{1 + x}$.
    1/2 and 4/3.
    The second version was easier to apply and the results agree.

23. (a) All real numbers; and all non-negative real numbers ($x \geq 0$).

    (b) True.

24. True.

25. $\sqrt{x^2} = |x|$.

    Reason: It was observed in the previous problem that the answer is the positive square root of the square of the absolute value of x. Since the absolute value of x is always non-negative, it is the positive square root of its square.

    The sign of the answer is non-negative and so was the original expression. It makes sense because the result surely had to have the same absolute value as the original. If x is positive, the answer has to also be positive and the answer equals x; but if x is negastive, then the answer has to be positive and that means the answer is $-x$. The abslute value accomplishes this exactly.

26. No! The correct answer is $a = \pm\frac{2}{3}\sqrt{2}$.

27. $a < 1$; $a^2 < a$; since the square root function is increasing, $\sqrt{a^2} < \sqrt{a}$; $\sqrt{a^2} = a$, because a is positive.

    That shows that $a < \sqrt{a}$.