Math 130 *Kovitz*

Some Rules for Factoring Polynomial Expressions

$A^2 + B^2$ has no simple factoring rule.

$$A^2 - B^2 = (A - B)(A + B)$$

$$A^2 + 2AB + B^2 = (A + B)^2$$

$$A^2 - 2AB + B^2 = (A - B)^2$$

Do not make the common mistake of saying that $A^2 - B^2$ is equal to $(A - B)^2$. That is not true.

One way of remembering the last two formulas is verbally.

The square of a binomial equals the first term squared plus the second term squared, plus or minus twice the product of the two terms. Don't forget the middle term.

To factor the difference of two squares, multiply the sum of the original numbers by the difference of the original numbers.

Problems.

1. Factor $a^2 + 64$. Answer: it is impossible.
2. Factor $49 - c^2$. Answer: $(7 - c)(7 + c)$.
3. Factor $a^{10} - b^{10}$. Answer: $(a^5 - b^5)(a^5 + b^5)$.
4. Factor $4a^2 - 12ab + 9b^2$. Answer: $(2a - 3b)^2$.
5. Factor $7 + 4\sqrt{3}$, which is equal to $4 + 4\sqrt{3} + 3$ (if that helps).
   Answer: $(2 + \sqrt{3})^2$.
6. Multiply $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$. Answer: $2 - 3$, which is just -1.
7. Square $\frac{1}{2}a - 2b$. Answer: $\frac{1}{4}a^2 - 2ab + 4b^2$.
8. Multiply out $(a^2 + b^2)(a + b)(a - b)$. Answer: $a^4 - b^4$.
9. Factor as a perfect square $8a^2 - 8a + 2$.
   Answer: $\left[2\sqrt{2}a - \sqrt{2}\right]^2 = \left[\sqrt{2}(2a - 1)\right]^2 = (\sqrt{2})^2(2a - 1)^2 = 2(2a - 1)^2$.
   As a final answer, the form without radicals is much preferred.