Cancelling in Fractions

Rule.

A term can be cancelled in a fraction if it is a *factor* of both the numerator and the denominator. Otherwise, it cannot be cancelled.

Examples.

1. $\dfrac{ab}{ac} = \dfrac{\cancel{a}b}{\cancel{a}c} = \dfrac{b}{c}$ — The term a is a factor of both the numerator and the denominator, so it cancels.

2. $\dfrac{a+b}{ac}$ — The term a cannot be cancelled, since it is not a factor of the numerator.

(The following steps are valid for this fraction:

$$\frac{a+b}{ac} = \frac{a}{ac} + \frac{b}{ac} = \frac{\cancel{a}}{\cancel{a}c} + \frac{b}{ac} = \frac{1}{c} + \frac{b}{ac}$$

but that is not a simple cancellation.)

3. $\dfrac{ab+ac}{ad} = \dfrac{a(b+c)}{ad} = \dfrac{\cancel{a}(b+c)}{\cancel{a}d} = \dfrac{b+c}{d}$ — The term a is a factor of both the numerator and the denominator, so it cancels.

4. $\dfrac{10t-2}{4} = \dfrac{2(5t-1)}{2\cdot 2} = \dfrac{\cancel{2}(5t-1)}{\cancel{2}\cdot 2} = \dfrac{5t-1}{2}$ — This is like Example 3. We can cancel 2.

Practice. Simplify the fractions by cancelling, when it is possible to cancel.

1. $\dfrac{xy}{zy}$
2. $\dfrac{2+x}{2x}$
3. $\dfrac{3-k}{3}$
4. $\dfrac{6+3k}{3}$
5. $\dfrac{4a-ta}{3a+ax}$
6. $\dfrac{4+2\sqrt{3}}{2}$
7. $\dfrac{4-\sqrt{6}}{2}$
8. $\dfrac{x}{x+y}$
9. $\dfrac{6-\sqrt{20}}{2}$

Answers.

1. $\dfrac{x}{z}$ (cancel y)
2. Cannot cancel (but it does $= \dfrac{1}{x} + \dfrac{1}{2}$, as in Example 2.)
3. Cannot cancel.
4. $2 + k$ (cancel 3)
5. $\dfrac{4-t}{3+x}$ (cancel a)
6. $2 + \sqrt{3}$ (cancel 2)
7. Cannot cancel.
8. Cannot cancel, or even rework as in Example 2.
9. $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$, so 2 cancels, giving $3 - \sqrt{5}$.