Rational Equation Example

Math 130 *Kovitz*

Solve for x.

$$4 = \frac{x^2}{x + 168}$$

Clear the fractions by multiplying each side of the equation by the denominator: $x + 168$. Then see if the resulting equation is linear or quadratic. If quadratic, decide on a method of solution. Then either isolate the x terms (if completing the square) or set the right side to 0 (if factoring or using the quadratic formula). *Do not* isolate the x terms on one side and divide both sides by the result of factoring out an x. That definitely will not be correct; it is not a linear equation, so methods for solving a linear equation are not applicable.

It is always a good idea to check the solutions in the *original* equation, using a calculator if necessary.

$4(x + 168) = \dfrac{x^2}{x + 168}\left(x + 168\right)$ Multiply each side by the denominator.

$4x + 672 = x^2$ Distribute on the left side; cancel factors on the right side.

$672 = x^2 - 4x$ Isolate the x terms. Completing the square looks very promising because the x^2 term has no coefficient and the x term has an even integer as its coefficient.

$x^2 - 4x = 672$ Switch sides of the equation to get a more familiar pattern.

$x^2 - 4x + 4 = 672 + 4$ Divide the coefficient of x by two, then square it.
That gives the completing number, 4, which is to be added to both sides.

$(x - 2)^2 = 26^2$ Write each side as a perfect square.

$x - 2 = \pm 26$ Take square roots of both sides introducing "plus or minus."

$x = 2 \pm 26$ Add 2 to both sides.

$x = 28$ or $x = -24$

Check the answers.

$$4 = \frac{28^2}{28 + 168} = \frac{784}{196} = 4.$$

and

$$4 = \frac{(-24)^2}{-24 + 168} = \frac{576}{144} = 4.$$

Both answers check.