Rational Equation Example
Math 130 *Kovitz*

Solve for x.

$$x^2 = \frac{2}{1 + \frac{1}{x}}$$

Clear the fractions by multiplying each side of the equation by the denominator: $1 + \frac{1}{x}$.

Then see if the resulting equation is linear or quadratic. If quadratic, decide on a method of solution.

Either isolate the x terms (if completing the square) or set the right side to 0 (if factoring or using the quadratic formula).

Do not isolate the x terms on one side and divide both sides by the result of factoring out an x. That definitely will not be correct; it is not a linear equation, so methods for solving a linear equation are not applicable.

It is always a good idea to check the solutions in the *original* equation, using a calculator if necessary.

$x^2 \left(1 + \frac{1}{x}\right) = \frac{2}{x + \frac{1}{x}} \left(x + \frac{1}{x}\right)$ Multiply each side by the denominator.

$x^2 + x = 2$ Distribute on the left side; cancel factors on the right side.

$x^2 + x - 2 = 0$ Completing the square looks complicated because the completing number will be a fraction. Set the right side of the equation to 0 and attempt to factor.

$(x + 2)(x - 1) = 0$ Easy trial and error.

$x = -2$ or $x = 1$

Check the answers in the original equation.

$$(-2)^2 = \frac{2}{1 + \frac{1}{-2}} = \frac{2}{1 + \left(-\frac{1}{2}\right)} = \frac{2}{1/2} = 4.$$

$$4 = 4.$$

and

$$1^2 = \frac{2}{1 + \frac{1}{1}} = \frac{2}{2} = 1.$$

$$1 = 1.$$

Both answers check.