Algebra Review

The Basics of Solving Quadratic Equations

1. Solve.
   (a) $3x^2 = 21$
   (b) $(x-1)^2 = 6$
   (c) $(x-13)^2 = 20$
   (d) $y^2 - 14y + 49 = 4$

Steps for solving a quadratic equation by completing the square.

- Clear out all fractions and all parentheses.
- Move the constant term to one side of the equation and all other terms to the other side.
- Divide both sides of the equation by the coefficient of x^2 (if it is a number other than 1).
- Take the coefficient of x, divide it by 2, square the result, and add that square to both sides of the equation.
- Factor one side of the equation into a perfect square and solve the equation. Taking the square root of both sides of the equation will introduce plus or minus in front of the number on the right side.

2. Solve by completing the square.
   (a) $x^2 + 4x = 2$
   (b) $x^2 - 22x = 11$
   (c) $x^2 - x = 3$
   (d) $x^2 + 10x - 4 = 0$

The Quadratic Formula

The solutions to the equation

$$ax^2 + bx + c = 0$$

are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Solve.

1. $x^2 - 6x - 4 = 0$
2. $h^2 + 6 = 10h$
3. $3x(x+1) - 7x(x+2) = 6$
4. $11(x-2) + x - 5 = (x+2)(x-6)$

Did you know that there is an alternate form of the quadratic formula? Is it ever helpful?

The solutions to the equation

$$ax^2 + bx + c = 0$$

are

$$x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}}.$$

Solve, using both versions of the quadratic formula. (Which was more convenient?)

1. $\left(\sqrt{3} + 1\right) x^2 - 3x + \sqrt{3} - 1 = 0$
2. $\sqrt{12}x^2 - 7x + \sqrt{3} = 0$ (also try completing the square for this example)

Guessing

Solve, by guessing one root, and finding the other root by means of the facts that the sum of the roots equals $\frac{-b}{a}$ and the product of the roots equals $\frac{c}{a}$.

1. $x^2 - \sqrt{2}x + \sqrt{2} - 1 = 0$
2. $x^2 - \sqrt{3}x + 2\sqrt{3} - 4 = 0$

The Discriminant and Solutions to Quadratic Equations

The solutions to the equation

$$ax^2 + bx + c = 0$$

are

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

The expression $b^2 - 4ac$ is called the **discriminant.** The value of the discriminant gives some information about the nature of the solutions. We summarize.

Discriminant	**Nature of solutions**
0	Only one real solution; generally best solved by factoring as a perfect square
0, with a, b, and c rational	Only one rational solution; solvable by factoring as a perfect square
Positive	Two different real-number solutions
A non-zero perfect square, with a, b, and c all rational	Two different rational solutions; solvable by factoring
Negative	No real-number solutions

1. Determine the nature of the solutions of the equation.

   (a) $x^2 + 22x + 121 = 0$

   (b) $x^2 + 16 = 0$

   (c) $x^2 - 7 = 0$

   (d) $4x^2 - 20x + 25 = 0$

   (e) $4x^2 + 12x - 7 = 0$

   (f) $x^2 - x + 9 = 0$

   (g) $9m^2 + 5m = 0$

2. With the help of the discriminant, determine the nature of the solutions. If there are any real-number solutions, find an efficient method and then use that method to solve the equation.
   (a) $4x^2 - 7x = -3$.
   (b) $5x^2 - 3x = 4$.
   (c) $7x^2 - 2x = -3$.
   (d) $9x^2 - 12x = -4$.
   (e) $x^2 - 10x = -11$.
   (f) $9x^2 - 9x + 1 = 0$.
   (g) $9x^2 - 12x + 1 = 0$.
   (h) $3x^2 - 9x + 5 = 0$.
3. Write a quadratic equation having the given numbers as a solution by using the principle of zero products in reverse.
   (a) $-3,\ 7$
   (b) -6, only solution
   (c) $-\frac{7}{9},\ \frac{2}{3}$

General Problems in Solving Quadratics

1. Solve, by any method you like, including graphing on a calculator.
   (a) $x^2 + 12x = 0$
   (b) $x^2 - \sqrt{5}x + \sqrt{5} - 1 = 0$
   (c) $3x^2 + 10x - 11 = 0$
   (d) $x^2 - 5x - 84 = 0$
   (e) $x^2 + x - 1 = 0$
   (f) $ax^2 + bx + c = 0, \quad$ with $a = \sqrt{2} + 1$, $b = 2$, and $c = \sqrt{2} - 1$.
2. Solve
   $$3x^2 - 17x + 10 = 0$$
   by using each of the six methods (factoring, completing the square, quadratic formula, alternate quadratic formula, guessing, and graphing on a calculator).
3. Which number has the property that if you take the product of the numbers 2 greater and 32 greater than it, you will obtain 50 times the original number as a result?