Quadratic Perfect Square Examples

Math 130 Kovitz

Solve for x.

$$\frac{9}{16}x^2 - x + \frac{4}{9} = 0.$$

Decide on a method of solution. Then either isolate the x terms (if completing the square) or set the right side to 0 (if factoring or using the quadratic formula).

Do not isolate the x terms on one side and divide both sides by the coefficient after factoring out an x. That's definitely not correct; it is not a linear equation, so methods for solving a linear equation are not applicable.

It is always a good idea to check the solutions in the original equation, using a calculator if necessary.

The given quadratic expression is a perfect square because $\left(\frac{3}{4}x\right)^2 = \frac{9}{16}x^2$, $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$, and the middle term is twice the product of $\frac{3}{4}x$ and $\frac{2}{3}$.

So rewrite as

$$\left(\frac{3}{4}x - \frac{2}{3}\right)^2 = 0.$$

Then

$$\frac{3}{4}x - \frac{2}{3} = 0.$$

This leads to $\frac{3}{4}x = \frac{2}{3}$, $x = (\frac{4}{3})(\frac{2}{3}) = \frac{8}{9}$.

Check the answer.

$$\frac{9}{16} \left(\frac{8}{9}\right)^2 - \frac{8}{9} + \frac{4}{9} = 0.$$

$$\frac{9}{16} \cdot \frac{64}{81} - \frac{4}{9} = 0.$$

$$\frac{4}{9} - \frac{4}{9} = 0.$$

It checks.

For similar examples, try:

$$\frac{9}{4}x^2 - x + \frac{1}{9} = 0.$$

and

$$\frac{1}{4}x^2 - x + 1 = 0.$$