Line Graphing Example

Math 130 *Kovitz*

Consider the straight line L through the point $(20, 1)$ with slope $= -0.1$.

1. Find an equation and both intercepts and sketch line L. Label the given point and both intercepts.
2. Draw a line segment connecting the origin and the midpoint M of the first-quadrant portion of line L. Is this line segment perpendicular to line L?
3. Find the length of the that line segment (that runs from the origin to point M).
4. Find the distance of the origin from line L.

ANSWERS

1. $y - 1 = -0.1(x - 20)$ — Put the point (x_1, y_1) and the slope m into the point-slope formula: $y - y_1 = m(x - x_1)$.

$y = 1 - 0.1x + 2 = -0.1x + 3.$ — For ease in finding the intercepts, solve for y. That gives slope-intercept form.

The y-intercept is $(0, 3)$. — In the slope-intercept form $y = mx + b$, the y-intercept is $(0, b)$.

$0 = -0.1x + 3$ — To find the x-intercept, set y to 0 and solve for x.

$0.1x = 3$

$x = 3/0.1 = \frac{3}{1/10} = 3 \times \frac{10}{1} = 30.$ — The x-intercept is $(30, 0)$.

2. $\mathrm{M} = \left(\frac{0+30}{2}, \frac{3+0}{2}\right) = (15, 1.5)$ — The midpoint formula is (average x, average y) $= \left(\frac{x_1+x_2}{2}, \frac{y-1+y_2}{2}\right)$.

$m = \frac{1.5}{15} = 0.1$ — Use the formula for the slope between $(0, 0)$ and $(15, 1.5)$, which is $\frac{\Delta x}{\Delta y}$.

The slope of line L was -0.1.

These are not negative reciprocals, so the lines are not perpendicular.

3. The line segment has length $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{15^2 + 1.5^2} \approx 15.0748$.
   The exact value is $\sqrt{15^2 + 1.5^2} = \sqrt{(10 \times 1.5)^2 + 1.5^2} = \sqrt{1.5^2(100 + 1)} = 1.5\sqrt{101}$.

4. An efficient way to solve this part involves the area of the triangle formed in the first quadrant by line L and the two axes.

   The distance d of this point to the line L is the length of a line segment containing the origin and perpendicular to line L. That line segment will be the height of the triangle if the line segment connecting the intercepts of line L is considered as the base of the triangle.

   Using b and h along the axes, the area will be $\frac{1}{2}bh = \frac{1}{2}30(3) = 45$, and the line segement connecting the intercepts of the triangle is found to be $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{30^2 + 3^2} = \sqrt{909} = 3\sqrt{101}$ long.

   Apply the area formula to the sides d and $3\sqrt{101}$.

$$45 = \frac{1}{2}d(3\sqrt{101}).$$

$$d = \frac{30}{\sqrt{101}} \approx 2.9851.$$

   It is also possible to find the exact point of intersection of the line $y = -0.1x + 3$ and the line with negative reciprocal slope that passes through the orgin, $y = 10x$. Solve the equation $10x = -0.1x + 3$. That gives the point $\left(\frac{3}{10.1}, \frac{3}{1.01}\right) \approx (.2940297, 2.970297)$.

   Its distance from the origin si $\sqrt{.2970297^2 + 2.972097^2} \approx 2.9851$.