Line Graphing Example

Math 130 *Kovitz*

Consider the straight line J through the point K $(2, 4.5)$ that has x-intercept 8. Call the x-intercept: point Q. Call the origin: point O.

1. Find an equation and both intercepts and sketch line J. Label the given point and both intercepts.
2. Let M be the midpoint of the first-quadrant portion of line J.
   Let N be the point on line J with $x = 6$.
   Plot M and N on the previous sketch, and label both of them with their coordiantes.
3. Let L be the point on line J that is closest to the origin.
   (a) Find the distance of the origin from line J (that will be the length of line segment OL).
   (b) Plot point L on the earlier sketch, and label it with both of its coordinates.
   (c) Find the distances of points K, M, and N from the origin; and show that each of them is bigger than the distance of the origin from point J.

ANSWERS

1. $\text{slope} = m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} =$ Point Q is at $(8, 0)$. Then find the slope of the given line J. Call it m.

$\dfrac{0-(4.5)}{8-2} = \dfrac{-4.5}{6} = \dfrac{-9}{12} = -\dfrac{3}{4} = -0.75.$

$y - 4.5 = -0.75(x-2)$ Put one of the points (x_1, y_1) and the slope m into the point-slope formula: $y - y_1 = m(x - x_1)$.

$y = 4.5 - 0.75x + 1.5 = -0.75x + 6.$ For ease in finding the intercepts, solve for y. That gives slope-intercept form.

The y-intercept is $(0, 6)$. In the slope-intercept form $y = mx + b$, the y-intercept is $(0, b)$.

2. $\text{M} = \left(\frac{0+8}{2}, \frac{6+0}{2}\right) = (4, 3)$ The midpoint formula is (average x, average y) $= \left(\frac{x_1+x_2}{2}, \frac{y-1+y_2}{2}\right)$.

$y = -0.75(6) + 6$ To find the second coordinate for point N, plug in $x = 6$ into the formula for J.

$y = -4.5 + 6 = 1.5$ Point N has coordinates $(6, 1.5)$.

3. (a) An efficient way to solve this part involves the area of the triangle formed in the first quadrant by line J and the two axes.
The distance d of this point to the line J is the length of a line segment containing the origin and perpendicular to line J. That line segment will be the height of the triangle if the line segment connecting the intercepts of line J is considered as the base of the triangle.
Using b and h along the axes, the area will be $\frac{1}{2}bh = \frac{1}{2}8(6) = 24$, and the line segement connecting the intercepts of the triangle is found to be $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ long.
Apply the area formula to the sides d and 10.

$$24 = \frac{1}{2}d(10).$$

$$d = \frac{48}{10} = 4.8.$$

(b) Point L is the exact point of intersection of the line $y = -\frac{3}{4}x + 6$ and the line with negative reciprocal slope that passes through the origin, $y = \frac{4}{3}x$. To find point L, solve the equation $\frac{4}{3}x = -\frac{3}{4}x + 6$.
$(\frac{4}{3} + \frac{3}{4})x = 6.$ $(\frac{16}{12} + \frac{9}{12})x = 6.$ $(\frac{25}{12})x = 6.$ $x = \frac{6\times 12}{25} = \frac{72}{25} = 2.88.$ Use $y = \frac{4}{3}x$ to get x.
$x = \left(\frac{4}{3}\right) 2.88 = 3.84.$
The coordinates of point L are $(2.88, 3.84)$.
Its distance from the origin is $\sqrt{2.88^2 + 3.84^2} = \sqrt{8.2944 + 14.7456} = \sqrt{23.04} = 4.8$, just as before.

(c) Distances: (use the fact that $\Delta x = x - 0 = x$ and $\Delta y = y - 0 = y$.)
$\text{K} = \sqrt{2^2 + 4.5^2} = \sqrt{24.25} \approx 4.9244 > 4.8.$ $\text{M} = \sqrt{4^2 + 3^2} = 5 > 4.8.$ $\text{N} = \sqrt{6^2 + 1.5^2} = \sqrt{38.25} \approx 6.1847 > 4.8.$