

Line Graphing Example

Math 130 Kovitz

Let J be the straight line through the points L: (4, 7) and N: (12, 5.4) (used for all 6 problems).

1. Find an equation and both intercepts and sketch line J. Label the two points given and the y -intercept as point C and the x -intercept as point D.
2. Find the length of the line segment LN, and find the exact coordinates of its midpoint M.
3. Find an equation of the perpendicular bisector (call it line K) of the line segment LN. Then graph it and label its intercepts and its point of intersection with line segment LN. Label the y -intercept of line K as point A, and label the x -intercept of line K as point B.
4. Find the area of triangle ACM.
5. Find the distance of the origin from line K. Designate the origin by point O.
(A hint: Find the length of line segment AB and then, by careful inspection of the picture, relate the area of triangle AOB to length AB and to the distance we are trying to find.)
6. Find the exact coordinates of the point on line K that is closest to the origin.
(You will need to know how to draw the shortest distance from a point to a line.)

ANSWERS

1. $\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$ First find the slope of the given line J. Call it m .

$$\frac{5.4 - 7}{12 - 4} = \frac{-1.6}{8} = -\frac{1}{5} = -0.2.$$

$$y - 7 = -0.2(x - 4)$$

Put one of the points (x_1, y_1) and the slope m into the point-slope formula:

$$y - y_1 = m(x - x_1).$$

$$y = 7 - 0.2x + 0.8 = -0.2x + 7.8.$$

For ease in finding the intercepts, solve for y . That gives slope-intercept form.

Point C, the y -intercept is (0, 7.8).

In the slope-intercept form $y = mx + b$, the y -intercept is $(0, b)$.

$$0 = -0.2x + 7.8$$

To find the x -intercept, set y to 0 and solve for x .

$$0.2x = 7.8$$

$$x = 7.8 / 0.2 = \frac{7.8}{1/5} = 7.8 \times \frac{5}{1} = 39.$$

Point D, the x -intercept is (39, 0).

2. $\text{Length} = \sqrt{8^2 + (-1.6)^2} = \sqrt{(5 \times 1.6)^2 + 1.6^2}$
 $= \sqrt{25(1.6)^2 + 1(1.6)^2} = \sqrt{(1.6)^2(26)} = 1.6\sqrt{26}$

The distance formula is $\sqrt{(\Delta x)^2 + (\Delta y)^2}$.

The length of LN is approximately 8.1584312.

$$M = \left(\frac{4 + 12}{2}, \frac{7 + 5.4}{2} \right) = (8, 6.2).$$

Formula is midpoint = (average x , average y).

3. $\text{slope} = -\frac{1}{-1/5} = 5.$

The slope of the perpendicular is the negative reciprocal of the original.

$$y - 6.2 = 5(x - 8)$$

Put point M (8, 6.2) and the slope 5 into the point-slope formula:

$$y - y_1 = m(x - x_1).$$

$$y = 6.2 + 5x - 40 = 5x - 33.8.$$

For ease in finding the intercepts, solve for y . That gives slope-intercept form.

Point A, the y -intercept is (0, -33.8). In the slope-intercept form $y = mx + b$, the y -intercept is (0, b).

$$0 = 5x - 33.8$$

To find the x -intercept, set y to 0 and solve for x .

$$5x = 33.8$$

$$x = 33.8/5 = 6.76.$$

Point B, the x -intercept is (6.76, 0).

4. For the triangle ACM: let AC be the base and it is oriented along the y -axis, hence vertical. Then the height is measured perpendicular to the base, hence in the horizontal direction. The horizontal distance from point M to the y -axis is precisely the first coordinate of M, so it is 8.

The base is equal to the distance between the points (0, 7.8) and (0, -33.8), so it equals

$$|y_2 - y_1| = 7.8 - (-33.8) = 41.6.$$

$$A = \frac{1}{2}bh = \frac{1}{2}(41.6)(8) = 4 \times 41.6 = 166.4.$$

5. AB has length $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{6.76^2 + 33.8^2} \approx 34.469372$.

Triangle AOB has area $\frac{1}{2}bh = \frac{1}{2}(6.76)(33.8) = 114.244$.

The distance from a point to a line is the length of a line segment from the point that is perpendicular to the line. That means that such a line segment is the height of triangle AOB, when AB is considered its base. Call that unknown distance d .

We have $A = \frac{1}{2}bh = \frac{1}{2}d(34.469372)$. Since A is known to be exactly 114.244, the equation reduces to $114.244 = \frac{1}{2}d(34.469372)$.

$$d = \frac{2(114.244)}{34.469372} \approx 6.628725.$$

6. The shortest distance from a point to a line is along a line that meets the original line at a right angle. In other words, drop a perpendicular from the point to the line. So from the origin, draw a line perpendicular to line K. It will have slope of $-1/5$, and the equation will be $y = -(1/5)x$. Now solve the two equations simultaneously to get the point of intersection.

$y = -(1/5)x$ means $x = -5y$. Plug it into $y = 5x - 33.8$ to get $y = 5(-5y) - 33.8$ and $26y = -33.8$. So $y = -33.8/26 = -1.3$. Then $x = -5y = -5(-1.3) = 6.5$.

The point of intersection is (6.5, -1.3).

There are three further checks that problem 5 was done correctly.

First: $d = \sqrt{6.5^2 + (-1.3)^2} = \sqrt{43.94} \approx 6.628725$.

Secondly: It is a curiosity that the lengths of line segments CM and LM are exactly the same. That means CM has length of 8.1584312, found in problem 2 for LN. Triangles AOK and ACM are similar, as they have the same angles. That means that the ratios of side CM to side AC in triangle ACM is equal to the ratio of side OK (the unknown) to side AO in triangle AOK.

That gives $\frac{x}{33.8} = \frac{8.1584312}{41.6}$ and $x = 33.8 \left(\frac{8.1584312}{41.6} \right) = 6.628725$.

A similarity argument could be made relating triangles AOK and ABO, but it is a bit more tricky since the right angle in ABO is now in the left side of the picture. The proportion is

$$\frac{x}{6.76} = \frac{33.8}{34.469372} \text{ and } x = 6.76 \left(\frac{33.8}{34.469372} \right) = 6.628725, \text{ as before.}$$