

## Warm-up Exercises on February 9

Math 130 *Kovitz* Spring 2016

1. Consider the expressions  $|a|$  and  $\sqrt{a^2}$ , where  $a$  is any real number.

True or false:

- Both of these expressions are defined only when  $a$  is not negative.
- Both of these expressions always yield a positive number or zero.
- The square of  $|a|$  is equal to  $a^2$ . If true, prove it.
- The square of  $\sqrt{a^2}$  is equal to  $a^2$ .
- If the squares of two positive numbers are equal, the numbers must be equal.

What does this prove about  $|a|$  and  $\sqrt{a^2}$ ?

2. Let  $(c, d)$  and  $(e, f)$  be any two points in the plane with  $c \neq e$ .

Prove that the straight line passing through them has a unique equation in the form  $y = mx + b$ .

3. Prove that the graph of any equation in the form  $y = mx + b$  is a straight line.
4. Verify that the slope of a straight line measures its steepness.
5. Prove that the line  $y = x$  bisects the right angle made by the positive  $x$ - and  $y$ -axes.
6. Prove that the shortest distance from an exterior point to a straight line is the length of the unique line segment containing the point and perpendicular to the line.
7. Verify, using slopes, that if a line is perpendicular to one of two parallel lines, it must be perpendicular to the other.
8. Start with any two parallel horizontal lines. Select any point on the upper line. Draw two rays from that point, each with angle of depression of 60 degrees from that point.

Show that the triangle formed by the two rays and the lower line is equilateral.