

Graphs of Equations

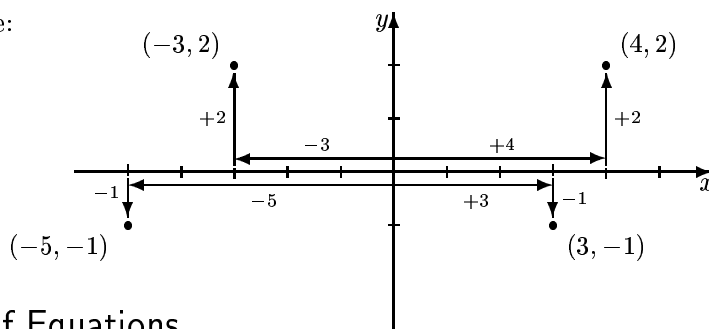
Math 130 Kovitz 2012

Plots of Ordered Pairs

To every ordered pair of real numbers there corresponds a unique point in the (real) coordinate plane. To find that point begin at the origin and first measure in a positive or negative horizontal direction (along the x -axis) an amount equal to the first member of the ordered pair, obtaining a new point. Then from that new point measure in a positive or negative vertical direction (parallel to the y -axis) an amount equal to the second member of the ordered pair, obtaining the desired point, the plot of the ordered pair of real numbers.

Similarly, given any point in the coordinate plane, we can find its coordinates by dropping a perpendicular to the x -axis and determining the directed height from the x -axis of that line and the directed length from the origin to the point where the perpendicular intersects the x -axis.

For example:



Graphs of Equations

If an equation has two variables, a solution is a pair of real numbers which, when substituted alphabetically, will yield a true equation. The solution set of the equation is the set of all ordered pairs that make the equation true.

For example, $y = 5x + 7$ has the ordered pair $(2, 17)$ as one of its solutions, since $17 = 5(2) + 7$. In fact, as is often the case, the equation $y = 5x + 7$ has infinitely many solutions—that is, infinitely many ordered pairs that make the equation true.

Now, if one simply plots in the xy -plane the ordered pairs which are the solutions to the equation, one obtains what is called the *graph* of the equation. The graph of an equation is a set of points or a *curve*. This fundamental idea is a most important concept in mathematics.

We might picture the idea in the following way as a series of correspondences:

ORDERED PAIR \longrightarrow POINT IN THE PLANE
 SOLUTION TO THE EQUATION \longrightarrow POINT ON THE GRAPH OF THE EQUATION
 SOLUTION SET OF THE EQUATION \longrightarrow GRAPH OF THE EQUATION
 EQUATION \longrightarrow SOLUTION SET \longrightarrow GRAPH

The left and middle columns consist of *algebraic* entities while the right column consists of *geometric* entities.

