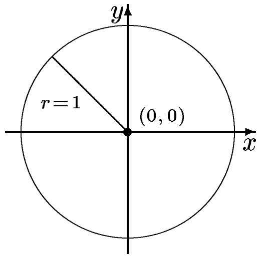
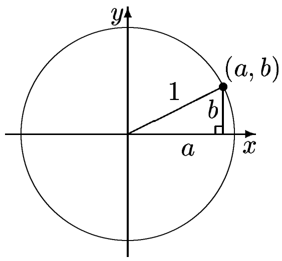
Notes on the Unit Circle

Math 130 *Kovitz*





The Unit Circle

The equation $x^2 + y^2 = 1$ produces a graph which is a circle with center at the point $(0,0)$ and radius 1.

Proof: Let (a,b) be an ordered pair in the solution set of $x^2 + y^2 = 1$. That means that $a^2 + b^2 = 1$. Draw the right triangle obtained when the point (a,b) is plotted in the plane. By the Pythagorean theorem $a^2 + b^2 = \text{hypotenuse}^2$. That means that the hypotenuse of this right triangle is 1. From this we conclude that the distance of the point (a,b) from the point $(0,0)$ is 1 because that distance is exactly the hypotenuse. We have thus shown that an arbitrary point in the solution set of $x^2 + y^2 = 1$ is on the circle with center at $(0,0)$ and radius 1.

Domain and Range

The domain of the unit circle (the set of all x-coordinates of points on the circle) is: $\{x| -1 \le x \le 1\}$.

The range of the unit circle (the set of all y-coordinates of points on the circle) is: $\{y| -1 \le y \le 1\}$.

Finding Points on the Unit Circle

Here are some examples of finding points on the unit circle.

To find the four intercepts $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$ we let $x = 1$, $y = 1$, $x = -1$, or $y = -1$. For example when $x = 1$, we have $1^2 + y^2 = 1$, $y^2 = 0$, and $y = 0$. From this we conclude that $(1,0)$ is the only point on the unit circle with $x = 1$. Similar methods will produce the other three intercepts.

When $x = .8$, we have $.8^2 + y^2 = 1$, yielding $.64 + y^2 = 1$, $y^2 = .36$, and $y = \pm .6$. The points on the unit circle when $x = .8$ are thus $(.8, .6)$ and $(.8, -.6)$.

It is easy to see that when $x = -.8$ the points on the unit circle are $(-.8, .6)$ and $(-.8, -.6)$.

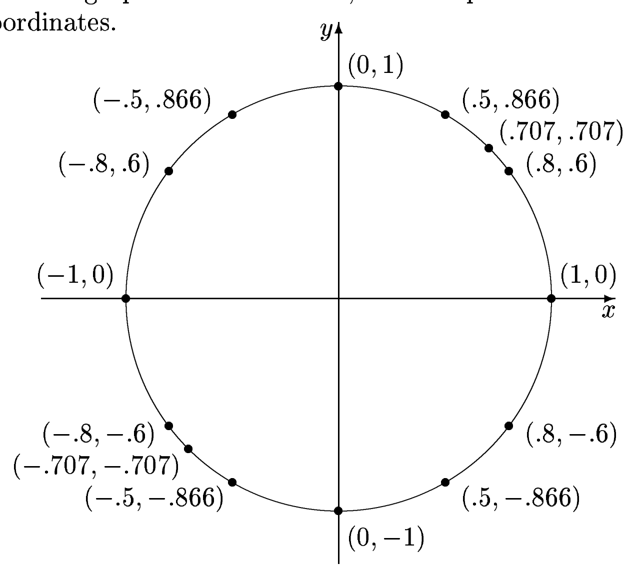
When $x = .5$, we have $.5^2 + y^2 = 1$, yielding $.25 + y^2 = 1$, $y^2 = .75 = 3/4$, and $y = \pm\sqrt{3/4} = \pm\sqrt{3}/2 \approx \pm 1.732/2 = \pm .866$. The points on the unit circle when $x = .5$ are thus approximately $(.5, .866)$ and $(.5, -.866)$.

It is easy to see that when $x = -.5$ the points on the unit circle are approximately $(-.5, .866)$ and $(-.5, -.866)$.

When $x = y$, substituting x for y in the equation $x^2 + x^2 = 1$ yields $2x^2 = 1$, $x^2 = 1/2$, and $x = \pm\sqrt{1/2} = \pm 1/\sqrt{2} = \pm(\frac{1}{\sqrt{2}})(\frac{\sqrt{2}}{\sqrt{2}}) = \pm\frac{\sqrt{2}}{2} \approx \pm\frac{1.414}{2} = \pm .707$. We also have $y = x$. This means that the points on the unit circle when $x = y$ are approximately $(.707, .707)$ and $(-.707, -.707)$.

The Graph of the Unit Circle

Here is the graph of the unit circle, with the points found so far labeled with their coordinates.



(Using $\frac{\sqrt{2}}{2} \approx .707$ and $\frac{\sqrt{3}}{2} \approx .866$.)