Practice on Completing the Square for Circles

For each problem:

- By completing the squares, put the equation into standard form.

- If the graph is a circle, find the center and the radius.

- Find the coordinates of the highest point on the graph of the circle.

- Find the exact coordinates of any x- and y-intercepts.

- Name the quadrants in which points on the graph lie.

1. $x^2 + y^2 + 4y = 5$.

2. $x^2 - x + y^2 + y + 1/4 = 0$.

3. $x^2 - 0.8x + y^2 + 10y = -4$.

4. $x^2 - 5x + y^2 - y = 2.5$.

5. $\frac{2}{3}x^2 + 10x + \frac{2}{3}y^2 - 4y = -19.5$.

6. $2x^2 - \sqrt{3}x + 2y^2 - 12y = -8.25$.

7. $x^2 - 2.4x + y^2 - 3.2y = -8$.

Answers follow.

1.
- $x^2 + (y + 2)^2 = 3^2$.
- The center is at $(0, -2)$, and the radius equals 3.
- The highest point on the graph has coordinates $(0, 1)$.
- The x-intercepts are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$; the y-intercepts are $(0, 1)$ and $(0, -5)$.
- There are points on the graph in all four quadrants.

2.
- $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$.
- The center is at $\left(\frac{1}{2}, -\frac{1}{2}\right)$, and the radius equals $\frac{1}{2}$.
- The highest point on the graph has coordinates $\left(\frac{1}{2}, 0\right)$.
- The x-intercept is $\left(\frac{1}{2}, 0\right)$; the y-intercept is $\left(0, -\frac{1}{2}\right)$.
- The fourth quadrant only.

3.
- $(x - 0.4)^2 + (y + 5)^2 = 4.6^2$.
- The center is at $(0.4, -5)$, and the radius equals 4.6.
- The highest point on the graph has coordinates $(0.4, -0.4)$.
- There are no x-intercepts; the y-intercepts are $(0, -5 + \sqrt{21})$ and $(0, -5 - \sqrt{21})$.
- The third and fourth quadrants.

4.
- $(x - 2.5)^2 + \left(y - \frac{1}{2}\right)^2 = 3^2$.
- The center is at $(2.5, 0.5)$, and the radius equals 3.
- The highest point on the graph has coordinates $(2.5, 3.5)$.
- The x-intercepts are $\left(\frac{5 + \sqrt{35}}{2}, 0\right)$ and $\left(\frac{5 - \sqrt{35}}{2}, 0\right)$; the y-intercepts are $\left(0, \frac{1 + \sqrt{11}}{2}\right)$ and $\left(0, \frac{1 - \sqrt{11}}{2}\right)$.
- There are points on the graph in all four quadrants.

5.
- $(x + 7.5)^2 + (y - 3)^2 = 6^2$.
- The center is at $(-7.5, 3)$, and the radius equals 6.
- The highest point on the graph has coordinates $(-7.5, 9)$.
- The x-intercepts are $(-7.5 + \sqrt{27}, 0)$ and $(-7.5 - \sqrt{27}, 0)$; there are no y-intercepts.
- The second and third quadrants.

6.
- $\left(x - \frac{\sqrt{3}}{4}\right)^2 + (y - 3)^2 = 2.25^2$.
- The center is at $\left(\frac{\sqrt{3}}{4}, 3\right)$, and the radius equals 2.25.
- The highest point on the graph has coordinates $\left(\frac{\sqrt{3}}{4}, 5.25\right)$.
- There are no x-intercepts; the y-intercepts are $\left(0, 3 + \frac{\sqrt{78}}{4}\right)$ and $\left(0, 3 - \frac{\sqrt{78}}{4}\right)$.
- The first and second quadrants.

7.
- $(x - 1.2)^2 + (y - 1.6)^2 = -4$.
- Over the real numbers: there are no solutions to this equation, the solution set is the empty set, and there is no graph.