f(x−h) AND f(x)

1. f(x−h) is a composite, or chain, of 2 functions: $\begin{cases} 1^{st} & (-h) : \text{subtract } h \\ 2^{nd} & (f) : \text{the given fcn} \end{cases}$

The rule for f(x−h):

diagram: $x \xrightarrow{(-h)} x-h \xrightarrow{(f)} f(x-h)$

or list the steps: $\begin{cases} x & \text{start} \\ x-h & \text{do } (-h) \\ f(x-h) & \text{do } (f) \end{cases}$

Examples to show the composite process.

a) Let $f(x) = x^2 + 3x$,
let $h = 2$,
compute the numbers in the table:

f(x)	x	x−h	f(x−h)
0	0	−2	−2
4	1	−1	−2
10	2	0	0
18	3	1	4
28	4	2	10
40	5	3	18

(2 step process)

b) Evaluate f(x−h) at a number called "x+h";
the result is f(x) because

$(x+h) \xrightarrow{(-h)} (x+h) - h = x \xrightarrow{(f)} f(x)$; or $f(x - h) \longrightarrow f((x+h) - h) = f(x)$ [plug in "x+h"]

2. The values of f(x−h) lag the values of f(x) by h, that is, a given value doesn't come out of f(x−h) until h units later (when $h > 0$).

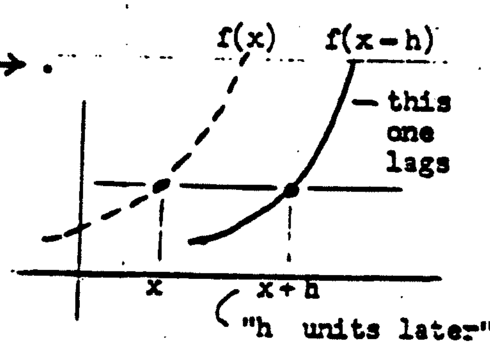
— in example a) the values lag by 2,
they come out 2 units later:

x	f(x)	f(x−2)
0	0	·
1	4	·
2	10	0
3	18	4
4	·	10
5	·	18

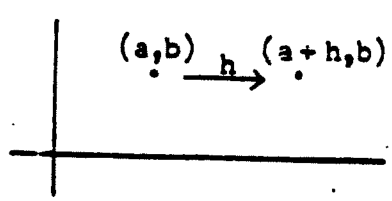
When $h < 0$ the values of f(x−h) lead those of f(x) by |h| units.
For ex. if $h = -3$ then the values of f(x+3) lead those of f(x) by 3 units.

3. The effect on graphs.

‖ The graph of f(x−h) is the graph of f(x) moved $\xrightarrow{h}$.



Proof. Suppose (a,b) is on the graph of f(x).
Then $b = f(a)$.
Now $(a+h, b)$ is on the graph of f(x−h),
because $f((a+h) - h) = f(a) = b$.
But $(a+h, b)$ is (a,b) moved $\xrightarrow{h}$.



to get the same height on the graph you have to increase x by h to compensate for (−h) which means $\xrightarrow{h}$.