

Transformation Examples

Math 130 Kovitz

We will illustrate transformations with an equation and the results of a few transformations applied to it.

Equation A:

$$y = x^2 - 4x + 10$$

Equation B:

$$y = (x + 3)^2 - 4(x + 3) + 10$$

1. What exactly was the replacement to get from A to B? What was put in, and for what letter?
 2. Does this replacement indicate that B is obtained from A by a translation, or a stretch/shrink?
 3. Is the transformation in the x or the y direction?
 4. Is the transformation horizontal or vertical?
 5. What is the numerical amount of the transformation? What distance is this?
 6. If you found that it is a translation, in which direction will this be (L, R, U, or D)? (*Be careful.*)
 7. Now, for conceptual clarity, look at the point on the graph of equation A for which $x = 5$. Call it P.
 - (a) Plug the coordinates of point P into equation A to show the equation is true.
 - (b) Q, the point on the graph of equation B with the same y as P, must have what coordinates?
 - i. Plug the coordinates of Q into equation B to show the equation is true. Why is this trivial?
 - ii. Put equation B into general form (without parentheses), and then plug point Q into the general form of equation B to show that the equation is true.
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Equation A:

$$y = x^2 - 4x + 10$$

Equation C:

$$y - 6 = x^2 - 4x + 10$$

Equation D:

$$y = (x^2 - 4x + 10) + 6$$

1. What exactly was the replacement to get from A to C? What was replaced, and for what letter?
2. Does this replacement indicate that C is obtained from A by a translation, or a stretch/shrink?
3. Is the transformation in the x or the y direction?
4. Is the transformation horizontal or vertical?
5. What is the numerical amount of the transformation? What distance is this?
6. If you found that it is a translation, in which direction will this be (L, R, U, or D)? (*Be careful.*)
7. Now, for conceptual clarity, look at the point on the graph of equation A for which $x = 5$. Call it P.
 - (a) Plug the coordinates of point P into equation A to show the equation is true. (done in 7(a) above)
 - (b) Point R on the graph of equation C with the same x as point P must have what coordinates?
 - i. Plug the coordinates of point R into equation C to show the equation is true.
 - ii. Show that equation D is just equation C in function form (solved for y), and plug point R into equation D to show that the equation is true. (Or, equivalently, plug R into $y = x^2 - 4x + 16$.)

Comparing equations A and D, we observe:

- Points P and R have the same the x -coordinate.
- Call the y -value of point P: y_{original}
- In equation D, there was no replacement for a letter; to the right side in function form was added 6.
- Call the y -value of point R: $y_{\text{translated}}$. So, $y_{\text{translated}} = (x^2 - 4x + 10) + 6 = y_{\text{original}} + 6$.
- The translated y was 6 more than the original y . That's all that happened.
- In this case: not a replacement, not opposite. So in what direction and how far was point P translated?

Rule for function form: whatever you apply to the whole right side will apply to y (direct, no opposite).