Even/Odd Problems

Math 130 *Kovitz*

1. In each case decide whether the function with the given rule is even, odd, or neither.

(a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

(b) $g(x) = \dfrac{x}{|x|} + x$

(c) $h(x) = -\dfrac{1}{x} - x$

(d) $i(x) = \dfrac{1}{x} - x$

(e) $j(x) = |x+1|$

(f) $f(x) = |x+3| - |x-3|$

(g) $g(x) = |x+3| + |x-3|$

(h) $h(x) = \dfrac{|x+3| - |x-3|}{|x+3| + |x-3|}$

(i) $i(x) = \sqrt{x+1} + \sqrt{1-x} + |x|$

(j) $j(x) = \dfrac{x^2 - 3x}{|x|}$

(k) $k(x) = \dfrac{1}{x^2 - 3} - 11x^6$

(l) $f(x) = \dfrac{1}{x} + 2x$

(m) $g(x) = x^3 - \dfrac{1}{x}$

(n) $h(x) = \dfrac{x^3 + x}{\sqrt[3]{x}}$

(o) $i(x) = \dfrac{x^4 - 1}{\sqrt[3]{x} - x}$

(p) $j(x) = |x-1|$

(q) $f(x) = \sqrt{x^2 + 2}$

(r) $g(x) = -|-x|$

(s) $h(x) = \sqrt{x-4}$. First determine the domain of h.

(t) $i(x) = \dfrac{x}{x-1} - \dfrac{x^2}{x-1}$. First determine the domain of i.

(u) $j(x) = \dfrac{1}{x^2 - 1} + \dfrac{1}{x+1}$.

Answers follow.

Answers.

1. (a) $f(x)$ is odd.
   (b) $g(x)$ is odd.
   (c) $h(x)$ is odd.
   (d) $i(x)$ is odd.
   (e) $j(x)$ is neither odd nor even.
   (f) $f(x)$ is odd.
   (g) $g(x)$ is even.
   (h) $h(x)$ is odd.
   (i) $i(x)$ is even.
   (j) $j(x)$ is neither odd nor even.
   (k) $k(x)$ is even.
   (l) $f(x)$ is odd.
   (m) $g(x)$ is odd.
   (n) $h(x)$ is even.
   (o) $i(x)$ is odd.
   (p) $j(x)$ is neither odd nor even.
   (q) $f(x)$ is even.
   (r) $g(x)$ is even.
   (s) $h(x)$ is neither odd nor even, because the domain $[4, \infty)$ is not balanced.
   (t) $i(x)$ must be neither odd nor even, because the domain (all reals except 1) is unbalanced. This is in spite of the fact that the formula is $i(x) = -x$ over the domain of definition. That tells us that $i(-x) = -(-x) = x = -i(x)$ whenever both x and $-x$ are in the domain of i. The case where $x = -1$ should lead to $i(-(-1)) = i(1)$, but $i(1)$ is not defined.

       The equality $i(-a) = -i(a)$ does not hold for all a in the domain of i, so the function is not odd.
   (u) $j(x)$ is odd. The domain is all reals except -1 and 1, which is balanced. The formula simplifies to $j(x) = \frac{x}{x^2-1}$, which is seen to be odd.

       If you are still skeptical, try finding $f(2)$ and $f(-2)$, getting $\frac{1}{4-1} + \frac{1}{2+1} = 2/3$ and $\frac{1}{(-2)^2-1} + \frac{1}{-2+1} = \frac{1}{3} + \frac{1}{-1} = \frac{1}{3} - 1 = -2/3$. For this choice, $-f(a)$ equals $f(-a)$, supporting the finding of odd.