Even and Odd Function Practice

Math 130 *Kovitz*

In each case decide whether the function f with the given rule is even, odd, or neither. Justify your answer.

1. $f(x) = c$
2. $f(x) = -x$
3. $f(x) = 2x^2 - 2x$
4. $f(x) = x^2 - 3x^4$
5. $f(x) = \sqrt[3]{x}\left(x^3 - x\right)$
6. $f(x) = \dfrac{x^2 - 1}{x}$
7. $f(x) = \dfrac{x^5 - x}{x^3}$
8. $f(x) = \dfrac{x^3 - x}{x - 1}$
9. $f(x) = |x - 1|$
10. $f(x) = 2 + \sqrt{x^2}$
11. $f(x) = \sqrt{x}$
12. $f(x) = (x - 1)^2 + 2x$

For an odd function with 0 in its domain, what can be concluded about the output to the function at 0?

Answers below

Answers with Justifications

1. Even. For all a: $f(-a) = f(a) = c$.
2. Odd. For all a: $f(-a) = -(-a) = a$, and $-f(a) = -(-a) = a$.
3. Neither. $f(-a) = 2a^2+2a$, but $f(a) = 2a^2-2a$ and $-f(a) = -2a^2+2a$.
4. Even. Rule of even powers.
5. Even. $f(-a) = \sqrt[3]{-a}\left((-a)^3 - (-a)\right) = [-\sqrt[3]{a}\,][-(a^3 - a)] = \sqrt[3]{a}(a^3 - a)$ and $f(a) = \sqrt[3]{a}(a^3 - a)$.
6. Odd. $f(-a) = \dfrac{a^2 - 1}{-a}$ and $-f(a) = -\left[\dfrac{a^2 - 1}{a}\right] = \dfrac{a^2 - 1}{-a}$.
7. Even. $f(-a) = \dfrac{(-a)^5 - (-a)}{(-a)^3} = \dfrac{-a^5 + a}{-a^3} = \dfrac{-(a^5 - a)}{-(a^3)} = \dfrac{-1}{-1} \cdot \dfrac{a^5 - a}{a^3} = \dfrac{a^5 - a}{a^3}$

   and $f(a) = \dfrac{a^5 - a}{a^3}$.
8. Neither. The domain includes -1 but not 1. Because its domain is unbalanced, the function is neither even nor odd.

   Alternatively, one could simplify as follows:

   $$x(x^2 - 1)/(x - 1) = x(x + 1)(x - 1)/(x - 1) = x(x + 1).$$

   From $f(2) = 6$ and $f(-2) = 2$, it is clear that the function is neither even nor odd.
9. Neither. $f(-a) = |-a-1| = |(-1)(a+1)| = |-1|\cdot|a+1| = 1\cdot|a+1| = |a+1|$, while $f(a) = |a - 1|$ and $-f(a) = -|a - 1|$.

   To show that the three expressions are not equivalent, choose a number in the domain of f, say 5, and plug it into all three expressions. The results (6, 4, and -4) would not be equal.
10. Even. Rule of even powers.
11. Neither. The domain includes all real numbers greater than or equal to 0. Because the domain is unbalanced, the function is neither even nor odd.
12. Even. Simplify the given expression $f(x) = (x^2 - 2x + 1) + 2x = x^2 + 1$, and then use the rule of even powers.

    Or, substitute to get $f(-a) = (-a - 1)^2 + 2a = (a^2 - 2a + 1) + 2a = a^2 + 1$, and substitute again to get $f(a) = (a - 1)^2 + 2a = (a^2 - 2a + 1) + 2a = a^2 + 1$.

If 0 is in its domain, any odd function must have output of 0 when 0 is the input. Otherwise, since the reflection through the origin would lead to input 0 and output of opposite sign, there would be two different outputs for the input of 0, contradicting the defintion of a function.

This principle could have been used in problems 1, 9, 10, and 12 to show that the functions were not odd.