Advanced Even and Odd Function Practice

Math 130 *Kovitz*

Decide whether the function f with the given rule is even, odd, or neither. Justify your answer.

1. $f(x) = (1-x)^{13} + (1+x)^{13}$.
2. $f(x) = (2+x)^{3/2} + (2-x)^{3/2}$.
3. $f(x) = (2+x)^{3/2} - (2-x)^{3/2}$.
4. $f(x) = x^{5/2} - x^{3/2}$.
5. $f(x) = x^{2/3} - x^{3/7}$.
6. $f(x) = x^{5/9} + x^{1/4}$.
7. $f(x) = x^{2/5} + x^{3/5}$.
8. $f(x) = x^{1/7} - x^{3/5}$.
9. $f(x) = x^{1/7} + x^{3/5}$.
10. $f(x) = x^{2/3} - x^{6/7}$.
11. $f(x) = \sqrt{|x+1| + |x-1|}$.
12. $f(x) = \sqrt[3]{|x+1| - |x-1|}$.
13. $f(x) = \sqrt{|x+1| - |x-1|}$.

Answers below

Answers with Justifications

1. Even. For all a: $f(-a) = (1+a)^{13} + (1-a)^{13} = f(a)$.

2. Even. For all a in the domain: $f(-a) = (2-a)^{3/2} + (2+a)^{3/2} = f(a)$.

3. Odd. For all a in the domain:
   $f(-a) = (2-a)^{3/2} - (2+a)^{3/2} = -[(2+a)^{3/2} - (2-a)^{3/2}] = -f(a)$.

4. Neither. The domain is all real numbers greater than or equal to 0. It is not balanced, so the function is neither even nor oddd.

5. Neither. For all a: $f(-a) = (-a)^{2/3} - (-a)^{3/7} = a^{2/3} + a^{3/7}$, but $f(a) = a^{2/3} - a^{3/7}$ and $-f(a) = -[a^{2/3} - a^{3/7}] = -a^{2/3} + a^{3/7}$.

6. Neither. The domain is all real numberes greater than or equal to 0 (otherwise the 4th root of x is not a real number). This doamin is not balanced, so the function is neither even nor odd.

7. Neither. For all a: $f(-a) = (-a)^{2/5} + (-a)^{3/5} = a^{2/5} - a^{3/5}$, but $f(a) = a^{2/5} + a^{3/5}$ and $-f(a) = -[a^{2/5} + a^{3/5}] = -a^{2/5} - a^{3/5}$.

8. Odd. For all a:
   $f(-a) = (-a)^{1/7} - (-a)^{3/5} = -a^{1/7} + a^{3/5} = -[a^{1/7} - a^{3/5}] = -f(a)$.

9. Odd. For all a: $f(-a) = (-a)^{1/7} + (-a)^{3/5} = -a^{1/7} - a^{3/5} = -[a^{1/7} + a^{3/5}] = -f(a)$.

10. Even. Foa all a: $f(-a) = (-a)^{2/3} - (-a)^{6/7} = a^{2/3} - a^{6/7} = f(a)$.

11. Even. For all a:
    $f(-a) = \sqrt{|-a+1| + |-a-1|} = \sqrt{|a-1| + |a+1|} = f(a)$.

12. Odd. For all a: $f(-a) = \sqrt[3]{|-a+1| - |-a-1|} = \sqrt[3]{|a-1| - |a+1|} = \sqrt[3]{-(|a+1| - |a-1|)} = -\sqrt{|a+1| - |a-1|} = -f(a)$.

13. Neither. It can be shown that the domain of f is $[0, \infty)$, which is unbalanced. To show that is rather difficult.

    Instead select a number a, and find $f(-a)$ and $f(a)$. For example: take 5. So $f(5) = \sqrt{2}$ and $f(-5)$ is undefined over the real numbers, because it is $\sqrt{-2}$. That is enough to show that neither $f(-a) = f(a)$ nor $f(-a) = -f(a)$ will hold for all real numbers in the domain of f, because 5 is in the domain of f and -5 is not. When $a = 5$ it neither of the two conditions can be met.