Advanced Even/Odd Problems

Math 130 *Kovitz*

1. Consider the function
$$f(x) = \sqrt{x+1} + \sqrt{x-1}.$$
Is the domain balanced? Use that result to determine if the function is even, odd, neither, or both.

   Verify your answer by analyzing $f(3)$ and $f(-3)$.

2. Consider the function
$$g(x) = \sqrt{x+1} + \sqrt{1-x}.$$
Find the domain. Is the domain balanced? Is that result sufficient to determine whether the function is even or odd?

   Is the function g even, odd, neither, or both?

   Support your answer by finding $g(1)$ and $g(-1)$.

3. Consider the function
$$h(x) = \sqrt{x+1} \cdot \sqrt{x-1}.$$
True or false: since an equivalent function is $f(x) = \sqrt{x^2-1}$, the function h must be an even function.

   Confirm your conclusion by looking at the domain of the function h and deciding if it is balanced or unbalanced.

   Use the results of $h(2)$ and $h(-2)$ to further support the conclusion.

4. Consider the function
$$g(x) = \sqrt{1+x} \cdot \sqrt{1-x}.$$
True or false: since an equivalent function is $f(x) = \sqrt{1-x^2}$, the function g must be an even function.

   Support your conclusion by looking at the domain of the function g and deciding if it is balanced or unbalanced.

   Compare $g(1)$ to $g(-1)$, $g\left(\frac{1}{2}\right)$ to $g\left(-\frac{1}{2}\right)$, and $g(0.96)$ to $g(-0.96)$ to further support the conclusion.

Answers follow.

Answers.

1. The domain is $[1, \infty)$; that is not balanced so the function must be neither.

   $f(3) = 2 + \sqrt{2}$, while $f(-3)$ is undefined.

2. The domain is $[-1, 1]$; that is balanced so no conclusion about odd or even may be drawn.

   $g(a) = \sqrt{a+1} + \sqrt{1-a}$ and
   $g(-a) = \sqrt{1-a} + \sqrt{1-(-a)} = \sqrt{1-a} + \sqrt{1+a} = g(a)$.
   It is an even function.

   $g(1) = \sqrt{2}$; $g(-1) = \sqrt{2}$. That tells us the function could well turn out to be an even function. It provides numerical support for the answer.

3. False. The functions are not equivalent since, for example, $h(-2) = \sqrt{3}$ while $h(-2)$ is not defined.

   The domain of h is $[1, \infty)$, which is not balanced. The function h must be neither even nor odd.

   $h(2) = \sqrt{3}$, but $h(-2)$ is not defined. By definition, it cannot be odd and it cannot be even.

4. True. The functions are equivalent, and f is even by the Rule of Even Powers.

   The domain of g is $[-1, 1]$, which is balanced. An even function must have a balanced domain. So far, the function g is potentially an even function, based on the domain.

   $g(1) = 0$, $g(-1) = 0$; $g\left(\frac{1}{2}\right) = \sqrt{3}/2$, $g\left(-\frac{1}{2}\right) = \sqrt{3}/2$; $g(0.96) = 0.28$, $g(-0.96) = 0.28$.

   In each case $g(-a) = g(a)$. For an even function that will be true for all a in the domain. Therefore, these three results support the conclusion.